

10 January, 2008

Norbert and Dennis,

In an earlier message, (E-mail on 12/19/07), in connection with the Mk.2A 6x97 matrix, I had written:

I do have some doubts about the integrity of the full 6x97 matrix since I see in it some terms that seem unreasonably large (and tend to cancel), and you have confirmed that your ANOVA results suggest some 'near singularities' when the load design is applied to the full 6x97 model. So I would agree that a matrix for which the math model does not show such linear dependencies would be a safer choice to use for a test. Whether the best choice is the "optimum matrix" as you have defined it, is something I'd like to discuss further with you - frankly I'm a bit disturbed to see terms being discarded purely on the basis of a statistical assessment of a single calibration dataset, and seemingly without consideration of the physical reasoning for their presence in the model in the first place. I intend to take a look at some other models, based on the term exclusions that you have identified but moderated by retaining (or rejecting) ALL terms that are 'related to each other' by the physical reasoning for their presence....

I will send you a fuller explanation of my concerns after I've had a chance to examine some results with other models

The phrase highlighted in red is what concerns me about the way in which BALFIT's math model term selection process is implemented, but I hasten to add that I am 100% in favour of the objective, namely the identification and removal of terms in the math model that are unsupported, or inadequately supported, by the calibration load design.

The basis of my concern lies in the fact that the 97-term equation, (i.e. 96 load terms + intercept), seemingly has come to be viewed as the fundamental representation of the balance's load/response relationship, whereas I consider the fundamental representation as being that expressed by Eqn. 3.1.2 in R-091-2003, namely:

$$R_i = a_i + \sum_{j=1}^n b_{i,j} F_j + \sum_{j=1}^n \sum_{k=j}^n c_{i,j,k} F_j F_k + \sum_{j=1}^n d_{i,j} F_j^3$$

This 'generally accepted' model comprises an intercept term and, for a 6-component balance, 33 'load' terms in F_j , F_j^2 , $F_j.F_k$, and F_j^3 . It can be made to account for possible asymmetric load effects by allowing each of the F_j , F_j^2 and F_j^3 terms to take one of two values according to the sign of the load F_j , and each of the $F_j.F_k$ terms to take 1 of 4 values according to the signs of the loads in the product $F_j.F_k$, (+/+, +/-, -/+ and -/-). Prior to the introduction of the 97-term math model representation in R-091-2003, multiple values for the various coefficients traditionally were determined for each load sign, or load sign combination, using procedures similar to the 'piece-wise curve-fitting' process described in Cook's paper, and also in my LR-600. Then, when calculating the loads equivalent to a particular vector of strain gauge bridge responses, the calibration matrix would be 'synthesized' by selecting its coefficients according to the signs of the loads, which initially were taken to be the same as the signs of the bridge responses. Occasionally this process would result in one or more of the computed loads having a sign different to that which had been used in the coefficient selection process, and in that case a new matrix would be selected based on the signs of the computed loads, and the loads calculation repeated. Such iteration was usually only required for loads that were close to zero, where interactions could lead to the load and the bridge response having different signs.

This is the model that essentially all of the members of the original AIAA 'Balance' WG already used and accepted. Some used a 6x27 model (no load cubed terms) instead of 6x33, and some determined coefficients as a function of load sign while some others did not, but I think all accepted that for some balances, (e.g. the 'Task' design), there were indeed potential slope discontinuities either side of zero load. In fact I recall Andy Garrell as saying that, because they (Calspan) needed to determine different coefficients based on the sign(s) of the loads, they did not use global regression because they couldn't determine the separate coefficients that way. **That was the point at which I pointed out that the asymmetric load behaviour could be modeled by the (now ubiquitous) 97-term equation to which global regression could be applied, and the 6x96 (or 6x97) matrix was born!!**

In hindsight I regret that, in what I wrote for the RP document, I did not place more emphasis on the fact that the function of the additional 'absolute value' terms is only to MODIFY the effect of the fundamental 'signed' terms. Consequently I consider it necessary that either ALL or NONE of the terms that modify a particular original 'signed' term must be present in the math model. Using the load product "N1.N2" as an example, the 'signed' term in the original 6x33 model is "N1.N2", and the three additional 'modifying' terms in the 97-term model are "|N1.N2|", "N1.|N2|" and "|N1|.N2". It is my opinion that, to properly account for asymmetric load behaviour, all 4 of these terms must be present in the math model since there is not a 1:1 relationship between the +/+, +/-, -/+

and +/- coefficients obtainable by a 'piece-wise curve-fitting' technique, and the coefficients of "N1.N2", "|N1.N2|", "N1.|N2|" and "|N1|.N2" in the 97-term math model. While the observed physical load/response characteristic of the balance, (that sensitivities to positive and negative load can be different), can be properly modeled by selecting specific coefficients from those for the + or -, or +/+, +/-, -/+ or -/-, signs of load or load combination, the same is not true for the 'absolute value' coefficients of the 97-term model. Only by utilizing ALL of the "N1.N2", "|N1.N2|", "N1.|N2|" and "|N1|.N2" terms together is the effect of a particular load product sign combination modeled correctly, by effectively selecting from the 4 possible 'N1.N2' load product coefficients the one that is appropriate to the particular signs of Fj and Fk. It is for this reason that I'm uneasy about the way in which BALFIT's math model search algorithm is implemented, since it allows each of the "N1.N2", "|N1.N2|", "N1.|N2|" and "|N1|.N2" terms to be assessed, and possibly excluded from the math model, independently of the others. I contend that, if any of those 4 terms are missing then the effects of load sign, i.e. sign of strain in the measuring elements, are not being modeled as the fundamental 6x33 math model intended. Of course, if no account is taken of asymmetric load behaviour, then only the 'signed' term, and NONE of the 3 terms involving 'absolute values', should be present in the math model, and in this case the 'signed' term can obviously be assessed on its own, both for proper definition by the loading design and significance to the result.

I would be most interested to see the 'math model term selection' results for the Mk.2A balance that would be produced if a modification were made to BALFIT's math model search algorithm to impose the following additional constraint. For each component in each of these 4 term groups, (not, I suggest, 10 separate 'groups' of terms as is frequently referred to in the AIAA papers you sent me),

$$\{ F_j \ \& \ |F_j| \}, \quad \{ F_j^2 \ \& \ F_j \cdot |F_j| \}, \quad \{ F_j \cdot F_k, \ |F_j \cdot F_k|, \ F_j \cdot |F_k| \ \& \ |F_j| \cdot F_k \}, \quad \{ F_j^3 \ \& \ |F_j^3| \}$$

the coefficients must either ALL BE PRESENT, or that ONLY the coefficients of the 'signed' terms, (Fj, Fj², Fj.Fk and Fj³), should be present, (assuming they are properly defined by the loading design and are statistically significant). Such a modification would reduce the number of models to be assessed, shortening the computation process.

Using the NASA Ames Mk.2A data I have made some experiments along these lines, starting with BALFIT's "optimum" {52, 42, 38, 37, 32, 50} matrix but then imposing the above type of constraint. I have tried six different matrices, all obtained by excluding and/or including terms (relative to the "optimum" model) in several combinations. Generally these matrices contain even fewer terms than BALFIT's "optimum" matrix, but for most of the components several of these modified matrices actually give lower error standard deviations for the Manual Check Loads and 6-comp. W/T Test Loads data files than the "optimum" matrix does. In the case of the back-calculation of the 'parent' calibration loads used to generate the matrices, (File 07), the error standard deviations are a measure of how well the selected model fits the data, and the matrix with the largest number of terms will usually give the lowest standard deviations. Consequently the standard deviations obtained with my modified matrices tend to be slightly higher than those obtained with the "optimum" matrix, and the full 6x97 matrix typically produces the lowest values. I think the results are sufficiently interesting to warrant further investigation.

A description of how the terms were selected for the six modified matrices, together with summarized statistics for nine different matrices, (6x97, 6x85, BALFIT's "optimum", and the 6 modifications based on the optimum), are given below. The results obtained for all matrices when computing the loads for each data file are initially shown on a single page, one page for each of files 07 (calibration data), 06 (manual loads data), and 03 (6-component W/T Test Loads data). Following that, the same results are presented at a larger scale with just one tabulation or plot per page. In the plots, the standard deviations are shown normalized by the smallest value and expressed as a percentage, i.e. the smallest standard deviation equals 100.

Finally, I should note that the standard deviation formula I have used employs the actual average error, and that the various statistics presented are based on ALL data points in each file, not just the 'non-zero load' points. I believe that a valid assessment of matrix performance should include all points and not just those points for which the load on the component under consideration is non-zero. Non-zero loads on the other components will cause interactions, and it is necessary to assess how well the matrix accounts for these. In my view 'zero' load is just another load, and so long as some of the components experience non-zero load I feel the error result for that condition should be included when assessing matrix performance. For points where ALL components are subjected to zero load, then there may be justification for excluding such points from the statistics so as to avoid giving undue weight to a condition which may be frequently repeated in some manually-loaded calibrations. However, for machine-loaded calibrations where load 'zeros' are unlikely to be either equal to zero or repeated precisely, I really see no need to exclude points that represent nominal zero load conditions for all components.

I will be most interested to have your feedback on my comments and the enclosed results.

Cheers, Robin

Matrix ID

Full 6 x 97 math model as in AIAA R-091-2003

6 x 85 math model = matrix "1" with the load cubed, (F³ and F³), terms excluded (set to zero)

Norbert Ulbrich's "optimum" matrix from BALFT's automatic term selection algorithm, terms/component = (52, 42, 38, 37, 32, 50)

Matrix with { 32, 28, 26, 14, 17, 27 } terms/component, based on matrix "3" but modified as follows:

- a) for the F, F², F², F², F³ and F³, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
 - b) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
 - c) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.
- Matrix with { 38, 26, 32, 24, 19, 31 } terms/component, based on matrix "3" but modified as follows:
- a) for the F and |F| terms, INCLUDE those terms for which both the "signed" and "absolute value" terms in "3" are zero,
 - b) for the F² and |F²| terms, INCLUDE any "signed" terms that are zero in "3", but for which the "absolute value" terms are non-zero,
 - c) for the F³ and |F³|, zero ALL terms,
 - d) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
 - e) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.

Matrix with { 36, 26, 30, 20, 15, 31 } terms/component, based on matrix "3" but modified as follows:

- a) for the F, |F| and F² terms, INCLUDE any "signed" terms for which the corresponding "absolute value" terms are non-zero,
- b) for the F³ and |F³|, zero ALL terms,
- c) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
- d) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.

Matrix with { 36, 28, 26, 22, 17, 27 } terms/component, based on matrix "3" but modified as follows:

- a) for the F, |F|, F², F², F³ and F³, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
- b) for the 2-load cross-product terms, set to zero the "absolute value" terms unless ALL 3 "absolute value" terms in "3" are non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero,
- c) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero,
- d) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.

Matrix with { 50, 37, 32, 33, 20, 47 } terms/component, based on matrix "3" but modified as follows:

- a) for the F, |F|, F², F², F³ and F³, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
- b) for the 2-load cross-product terms, set to zero all "absolute value" terms unless AT LEAST 2 "absolute value" terms in "3" are non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero.

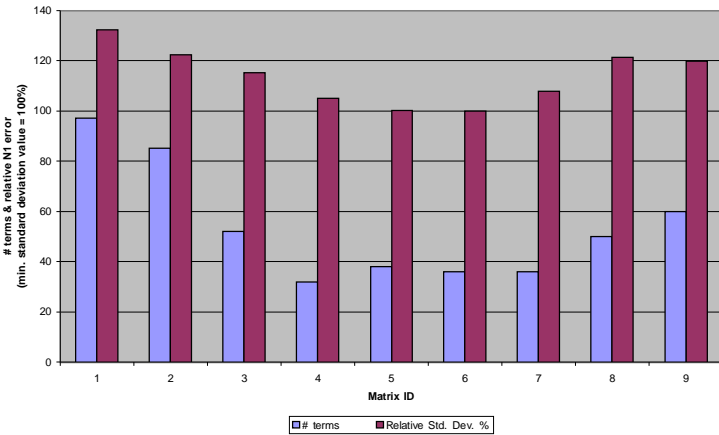
Matrix with { 60, 54, 50, 54, 60, 63 } terms/component, based on matrix "3" but modified as follows:

- a) for the F, |F|, F², F², F³ and F³, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
- b) for the 2-load cross-product terms, set to zero the "absolute value" terms unless AT LEAST 1 "absolute value" terms in "3" is non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero.

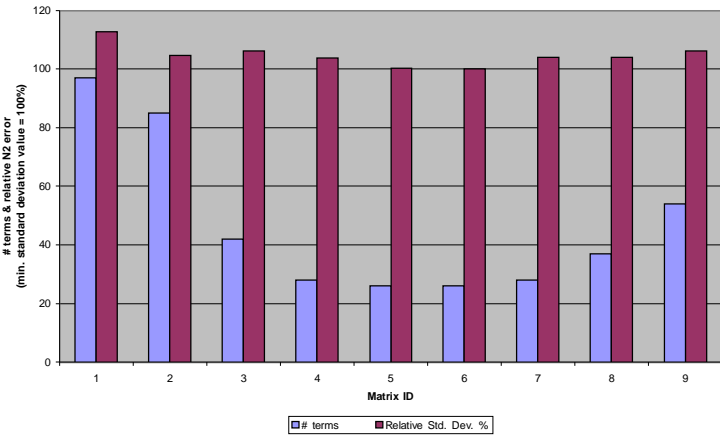
Error statistics summary from loads calculation of the Mk.2A generic 6-component W/T Test Loads File 03 using different matrices.

| Matrix ID | Mk error % | N1 error % | N2 error % | Y1 error % | Y2 error % | RM error % | AF error % |
|--|------------|------------|------------|------------|------------|------------|------------|
| Matrix "1" (6x97) | 0.023 | 0.103 | 0.063 | 0.224 | 0.067 | 0.313 | 0.253 |
| Matrix "2" (6x85) | 0.077 | 0.088 | 0.034 | 0.211 | 0.433 | 0.418 | 0.253 |
| Matrix "3" (Norbert Ulbrich "optimum") | 0.073 | 0.073 | 0.073 | 0.269 | 0.264 | 0.228 | 0.228 |
| Matrix "4" (Mod A of "optimum") | 0.146 | 0.031 | 0.204 | 0.269 | 0.263 | 0.158 | 0.158 |
| Matrix "5" (Mod A3 of "optimum") | 0.073 | 0.069 | 0.201 | 0.251 | 0.223 | 0.223 | 0.223 |
| Matrix "6" (Mod A of "optimum") | 0.431 | 0.104 | 0.067 | 0.192 | 0.161 | 0.359 | 0.359 |
| Matrix "7" (Mod B of "optimum") | 0.073 | 0.069 | 0.201 | 0.251 | 0.223 | 0.223 | 0.223 |
| Matrix "8" (Mod C of "optimum") | 0.468 | 0.073 | 0.069 | 0.238 | 0.205 | 0.417 | 0.417 |
| Matrix "9" (Mod D of "optimum") | 0.121 | 0.108 | 0.443 | 0.252 | 0.198 | 0.235 | 0.235 |

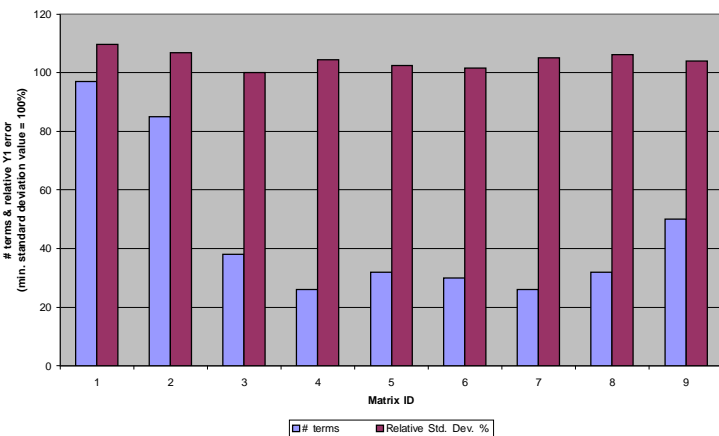
Mk.2A balance, N1 error relative standard deviations for 6-component generic W/T Test Loads (File 03)



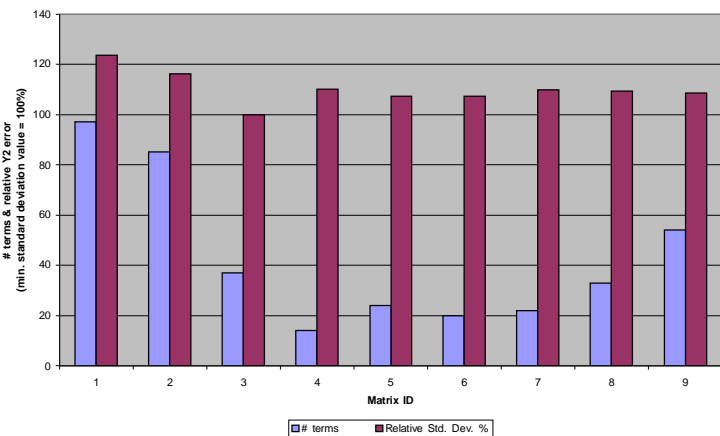
Mk.2A balance, N2 error relative standard deviations for 6-component generic W/T Test Loads (File 03)



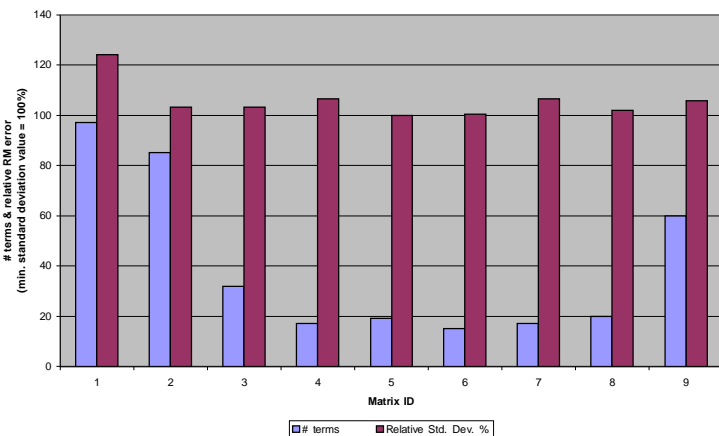
Mk.2A balance, Y1 error relative standard deviations for 6-component generic W/T Test Loads (File 03)



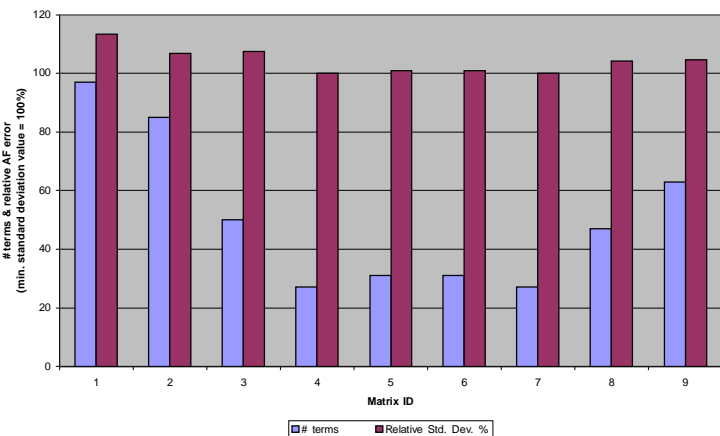
Mk.2A balance, Y2 error relative standard deviations for 6-component generic W/T Test Loads (File 03)



Mk.2A balance, RM error relative standard deviations for 6-component generic W/T Test Loads (File 03)



Mk.2A balance, AF error relative standard deviations for 6-component generic W/T Test Loads (File 03)



Matrix ID

- 1 Full 6 x 97 math model as in AIAA R-091-2003
- 2 6 x 85 math model = matrix "1" with the load cubed, (F^3 and $|F^3|$), terms excluded (set to zero)
- 3 Norbert Ulbrich's "optimum" matrix from BALFIT's automatic term selection algorithm, terms/component = {52, 42, 38, 37, 32, 50}
- 4 Matrix with { 32, 28, 26, 14, 17, 27 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F, |F|, F^2 , F.|F|, F^3 and $|F^3|$, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
 - b) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
 - c) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.
- 5 Matrix with { 38, 26, 32, 24, 19, 31 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F and |F| terms, INCLUDE those terms for which both the "signed" and "absolute value" terms in "3" are zero,
 - b) for the F^2 and F.|F| terms, INCLUDE any "signed" terms that are zero in "3", but for which the "absolute value" terms are non-zero,
 - c) for the F^3 and $|F^3|$, zero ALL terms,
 - d) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
 - e) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.
- 6 Matrix with { 36, 26, 30, 20, 15, 31 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F^2 and F.|F| terms, INCLUDE any "signed" terms that are zero in "3", but for which the "absolute value" terms are non-zero,
 - b) for the F^3 and $|F^3|$, zero ALL terms,
 - c) for the 2-load cross-product terms, set to zero all "absolute value" terms for which the corresponding "signed" product term in "3" is zero,
 - d) for the 2-load cross-product terms, if the "signed" product term in "3" is non-zero then set to zero all "absolute value" terms if any of those 3 terms is zero.
- 7 Matrix with { 36, 28, 26, 22, 17, 27 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F, |F|, F^2 , F.|F|, F^3 and $|F^3|$, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
 - b) for the 2-load cross-product terms, set to zero the "absolute value" terms unless ALL 3 "absolute value" terms in "3" are non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero.
- 8 Matrix with { 50, 37, 32, 33, 20, 47 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F, |F|, F^2 , F.|F|, F^3 and $|F^3|$, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
 - b) for the 2-load cross-product terms, set to zero the "absolute value" terms unless AT LEAST 2 "absolute value" terms in "3" are non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero.
- 9 Matrix with { 60, 54, 50, 54, 60, 63 } terms/component, based on matrix "3" but modified as follows:
 - a) for the F, |F|, F^2 , F.|F|, F^3 and $|F^3|$, set to zero all "absolute value" terms for which the corresponding "signed" term in "3" is zero,
 - b) for the 2-load cross-product terms, set to zero the "absolute value" terms unless AT LEAST 1 "absolute value" terms in "3" is non-zero, in which case INCLUDE ALL 4 cross-product terms, even if the "signed" cross-product term in "3" is zero.

Error statistics summary from loads calculation of the Mk.2A Calibration Loads File 07 using different matrices.

| Matrix "1" (6x97) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.362 | 1.238 | 0.584 | 2.282 | 4.056 | 0.379 |
| MIN error %FS | -0.472 | -0.999 | -0.770 | -2.517 | -3.023 | -0.384 |
| Absolute largest error %FS | 0.472 | 1.238 | 0.770 | 2.517 | 4.056 | 0.384 |
| Std. Devn. %FS | 0.066 | 0.274 | 0.145 | 0.651 | 0.663 | 0.114 |
| Average error %FS | 0.000 | 0.000 | 0.000 | -0.003 | 0.000 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 7.13 | 4.52 | 5.32 | 3.86 | 6.11 | 3.36 |
| # terms | 97 | 97 | 97 | 97 | 97 | 97 |

| Matrix "2" (6x85) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.430 | 1.153 | 0.597 | 2.186 | 4.217 | 0.371 |
| MIN error %FS | -0.437 | -0.982 | -0.752 | -2.340 | -3.240 | -0.379 |
| Absolute largest error %FS | 0.437 | 1.153 | 0.752 | 2.340 | 4.217 | 0.379 |
| Std. Devn. %FS | 0.070 | 0.272 | 0.147 | 0.647 | 0.662 | 0.114 |
| Average error %FS | 0.000 | 0.000 | 0.000 | -0.002 | -0.001 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 6.28 | 4.24 | 5.12 | 3.62 | 6.37 | 3.32 |
| # terms | 85 | 85 | 85 | 85 | 85 | 85 |

| Matrix "3" (Norbert Ulbrich "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.404 | 1.207 | 0.521 | 2.196 | 4.222 | 0.376 |
| MIN error %FS | -0.414 | -1.058 | -0.713 | -2.351 | -3.384 | -0.404 |
| Absolute largest error %FS | 0.414 | 1.207 | 0.713 | 2.351 | 4.222 | 0.404 |
| Std. Devn. %FS | 0.073 | 0.278 | 0.163 | 0.696 | 0.671 | 0.116 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.003 | -0.001 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 5.69 | 4.34 | 4.37 | 3.38 | 6.29 | 3.49 |
| # terms | 52 | 42 | 38 | 37 | 32 | 50 |

| Matrix "4" (Mod-A of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.481 | 1.355 | 0.593 | 2.514 | 5.175 | 0.531 |
| MIN error %FS | -0.390 | -1.032 | -0.623 | -2.995 | -3.325 | -0.437 |
| Absolute largest error %FS | 0.481 | 1.355 | 0.623 | 2.995 | 5.175 | 0.531 |
| Std. Devn. %FS | 0.087 | 0.295 | 0.180 | 0.827 | 0.763 | 0.142 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.003 | 0.000 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 5.54 | 4.60 | 3.46 | 3.62 | 6.78 | 3.73 |
| # terms | 32 | 28 | 26 | 14 | 17 | 27 |

| Matrix "5" (Mod-A2 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.543 | 1.251 | 0.581 | 2.256 | 4.703 | 0.504 |
| MIN error %FS | -0.362 | -0.915 | -0.634 | -2.715 | -3.312 | -0.426 |
| Absolute largest error %FS | 0.543 | 1.251 | 0.634 | 2.715 | 4.703 | 0.504 |
| Std. Devn. %FS | 0.085 | 0.295 | 0.160 | 0.779 | 0.758 | 0.136 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.004 | 0.000 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 6.37 | 4.24 | 3.96 | 3.48 | 6.20 | 3.72 |
| # terms | 38 | 26 | 32 | 24 | 19 | 31 |

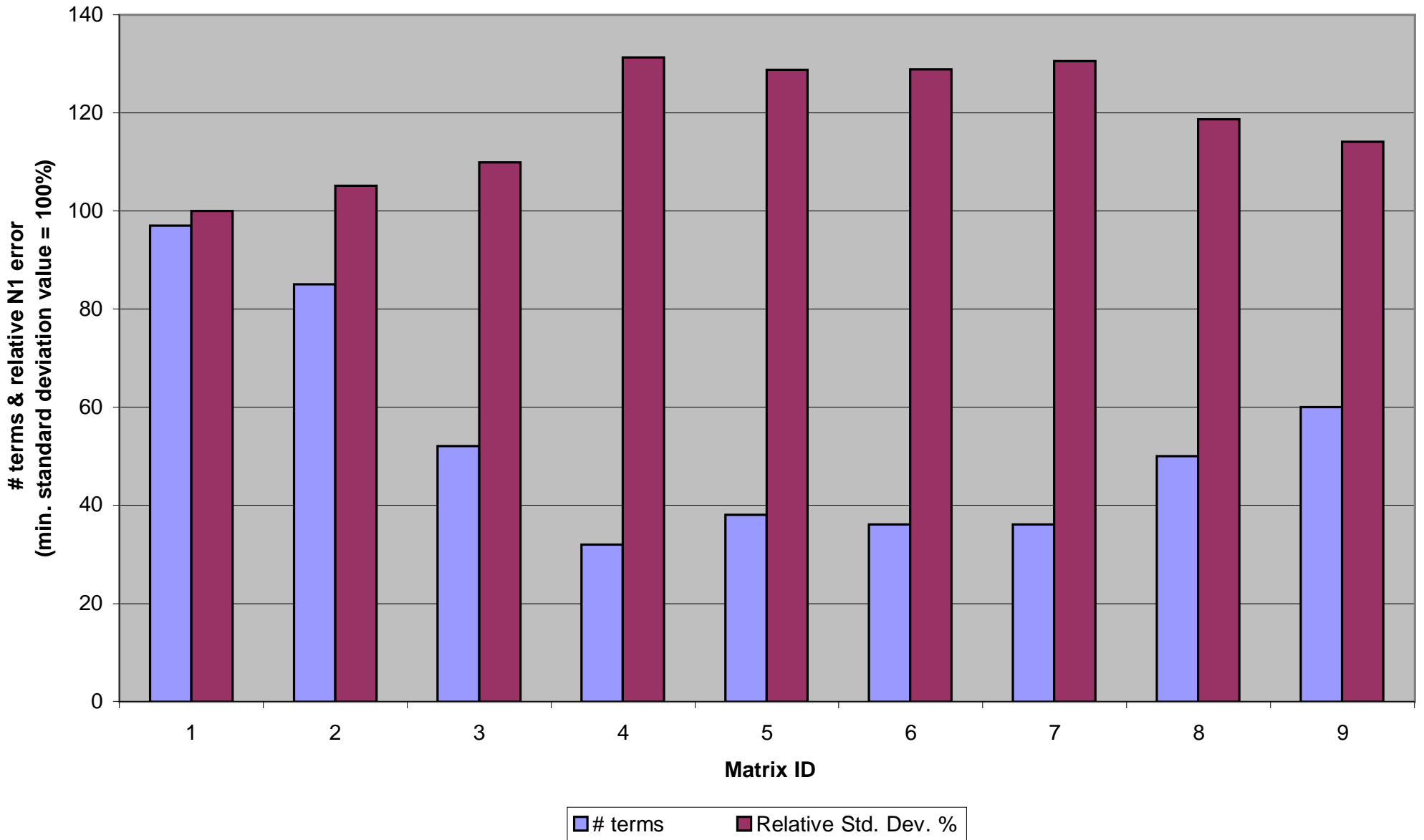
| Matrix "6" (Mod-A3 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.545 | 1.249 | 0.584 | 2.195 | 4.785 | 0.504 |
| MIN error %FS | -0.362 | -0.915 | -0.610 | -2.925 | -3.318 | -0.426 |
| Absolute largest error %FS | 0.545 | 1.249 | 0.610 | 2.925 | 4.785 | 0.504 |
| Std. Devn. %FS | 0.085 | 0.295 | 0.167 | 0.796 | 0.763 | 0.136 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.002 | 0.000 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 6.39 | 4.23 | 3.65 | 3.67 | 6.27 | 3.72 |
| # terms | 36 | 26 | 30 | 20 | 15 | 31 |

| Matrix "7" (Mod-B of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.487 | 1.357 | 0.610 | 2.424 | 5.175 | 0.531 |
| MIN error %FS | -0.390 | -1.033 | -0.617 | -2.816 | -3.330 | -0.437 |
| Absolute largest error %FS | 0.487 | 1.357 | 0.617 | 2.816 | 5.175 | 0.531 |
| Std. Devn. %FS | 0.086 | 0.295 | 0.180 | 0.801 | 0.763 | 0.142 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.003 | -0.001 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 5.64 | 4.61 | 3.43 | 3.52 | 6.78 | 3.73 |
| # terms | 36 | 28 | 26 | 22 | 17 | 27 |

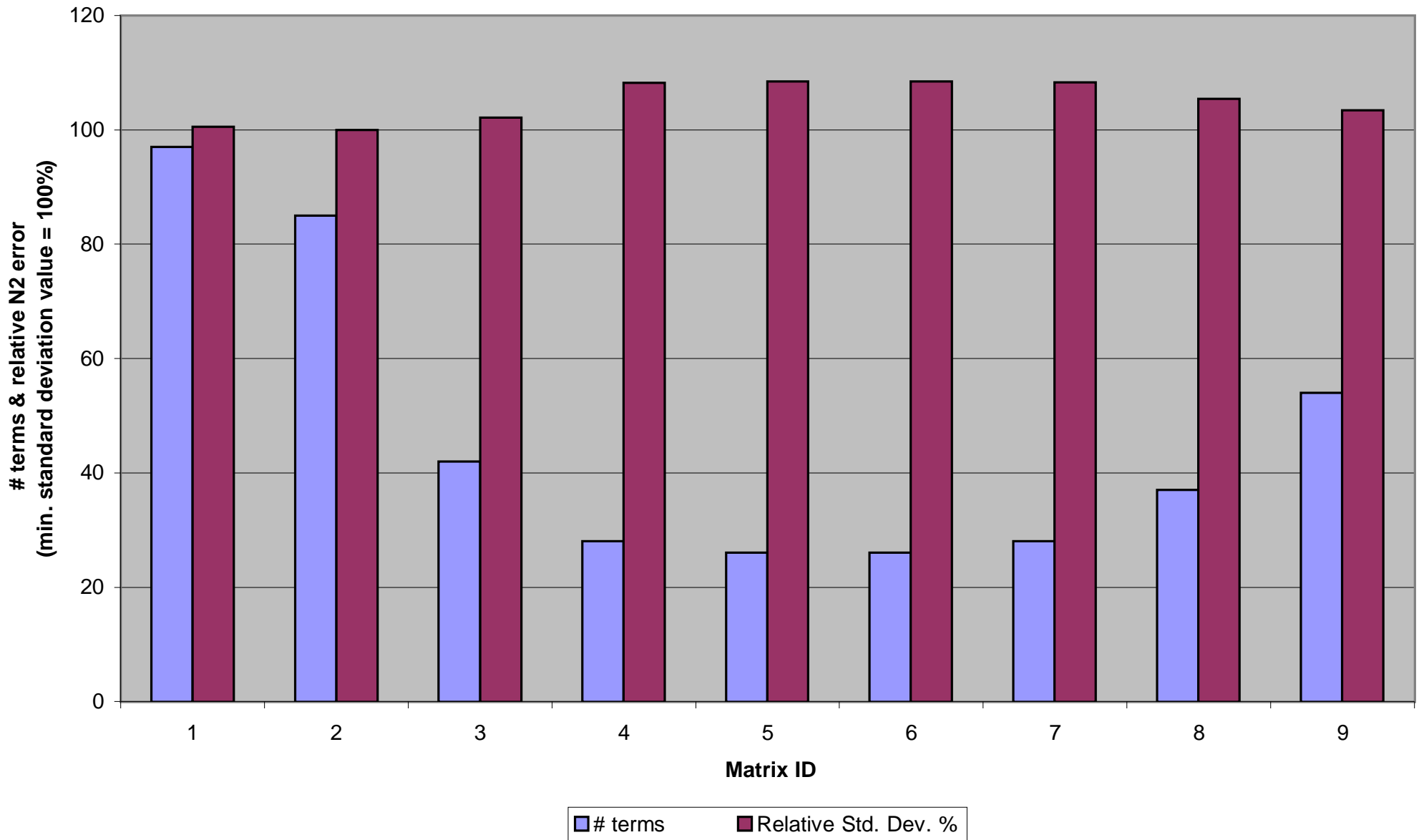
| Matrix "8" (Mod-C of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.477 | 1.181 | 0.683 | 2.352 | 4.083 | 0.400 |
| MIN error %FS | -0.423 | -1.113 | -0.700 | -2.684 | -3.441 | -0.434 |
| Absolute largest error %FS | 0.477 | 1.181 | 0.700 | 2.684 | 4.083 | 0.434 |
| Std. Devn. %FS | 0.079 | 0.287 | 0.175 | 0.758 | 0.705 | 0.122 |
| Average error %FS | 0.000 | 0.000 | -0.001 | -0.003 | -0.001 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 6.07 | 4.12 | 4.01 | 3.54 | 5.79 | 3.56 |
| # terms | 50 | 37 | 32 | 33 | 20 | 47 |

| Matrix "9" (Mod-D of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.340 | 1.178 | 0.658 | 2.237 | 4.268 | 0.404 |
| MIN error %FS | -0.422 | -1.061 | -0.697 | -2.379 | -3.373 | -0.442 |
| Absolute largest error %FS | 0.422 | 1.178 | 0.697 | 2.379 | 4.268 | 0.442 |
| Std. Devn. %FS | 0.076 | 0.281 | 0.170 | 0.708 | 0.668 | 0.121 |
| Average error %FS | 0.000 | 0.000 | 0.000 | -0.003 | -0.001 | 0.000 |
| Ratio of absolute largest error to Std. Devn. | 5.59 | 4.19 | 4.11 | 3.36 | 6.39 | 3.66 |
| # terms | 60 | 54 | 50 | 54 | 60 | 63 |

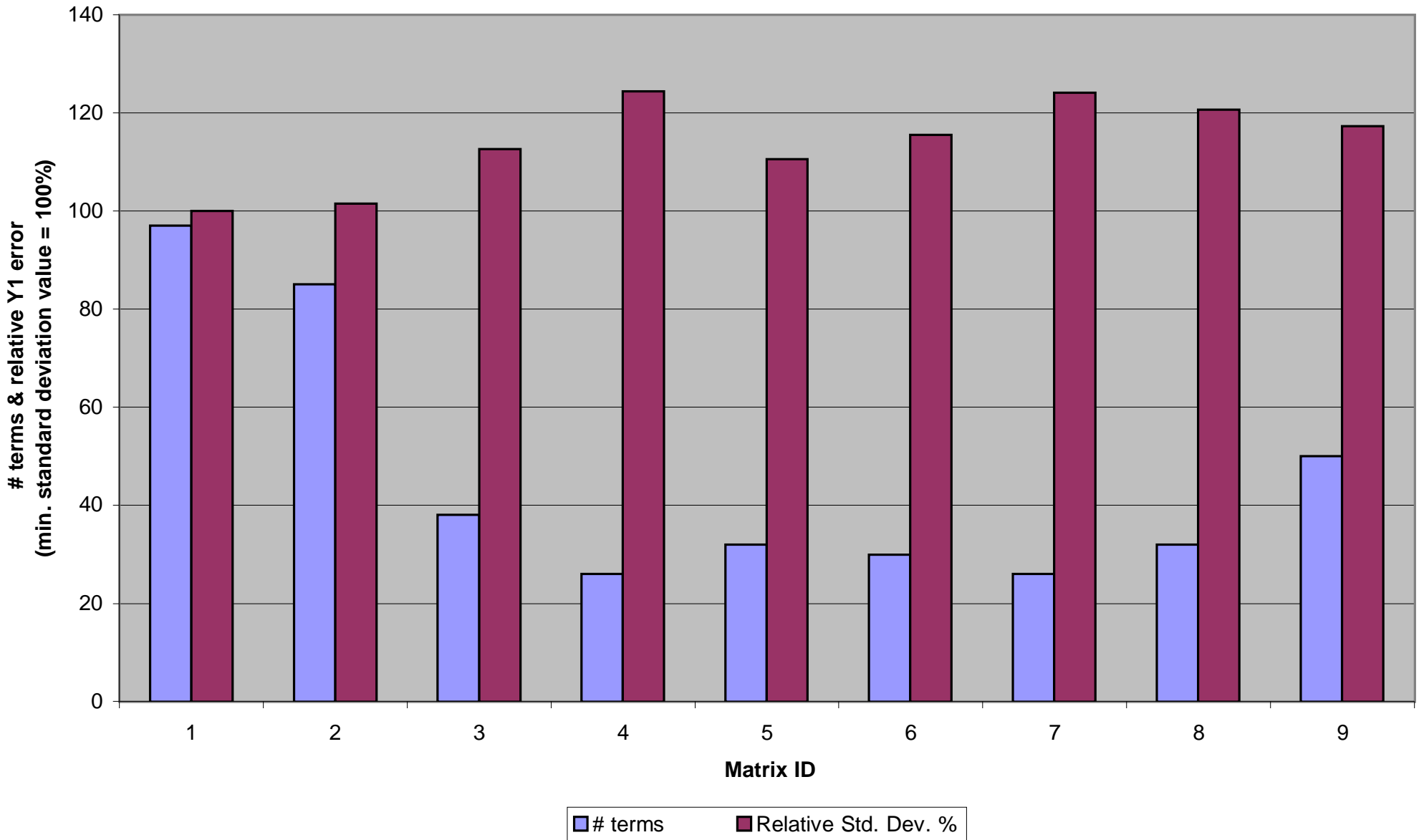
Mk.2A balance, N1 error relative standard deviations for
Calibration Loads (File 07)



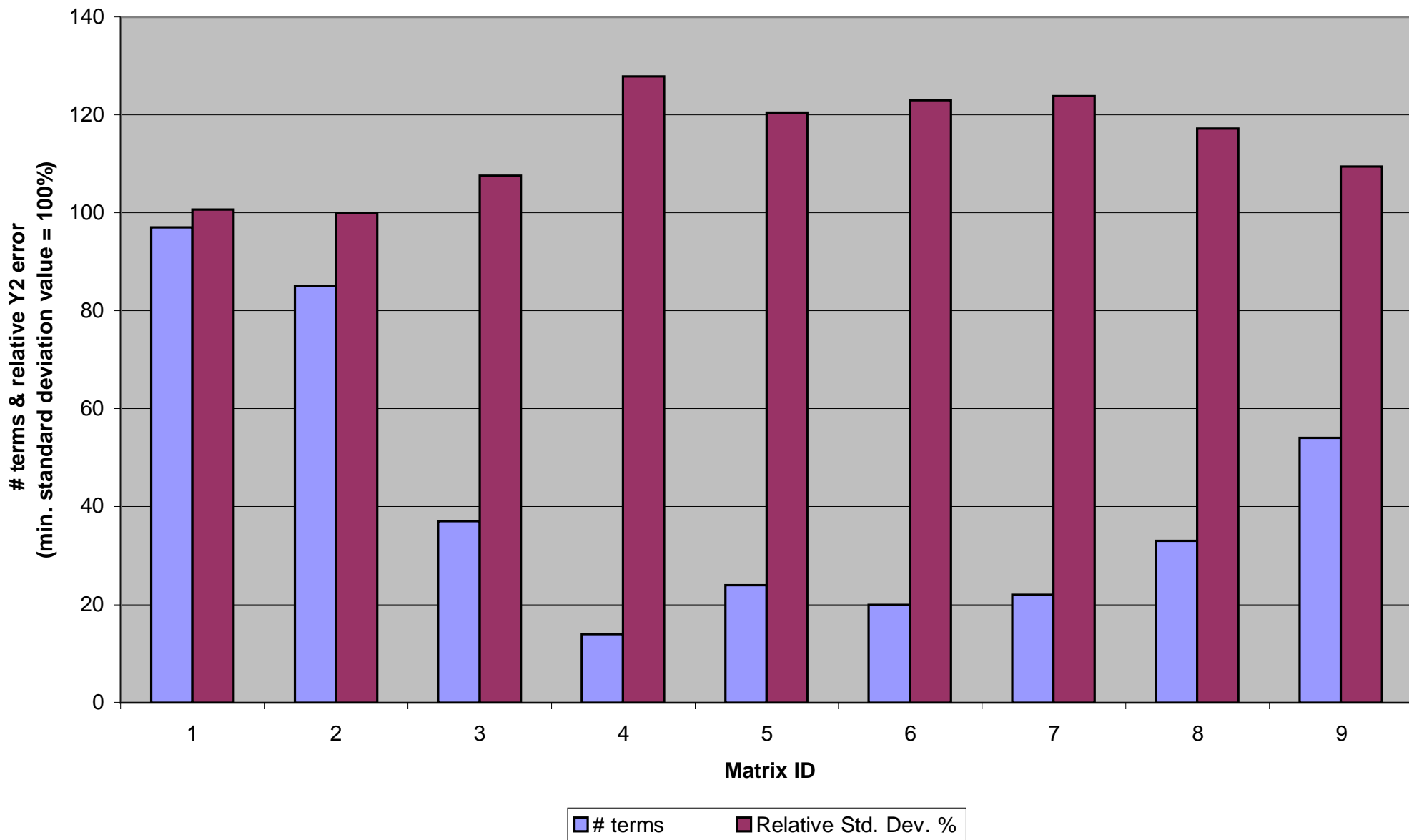
Mk.2A balance, N2 error relative standard deviations for
Calibration Loads (File 07)



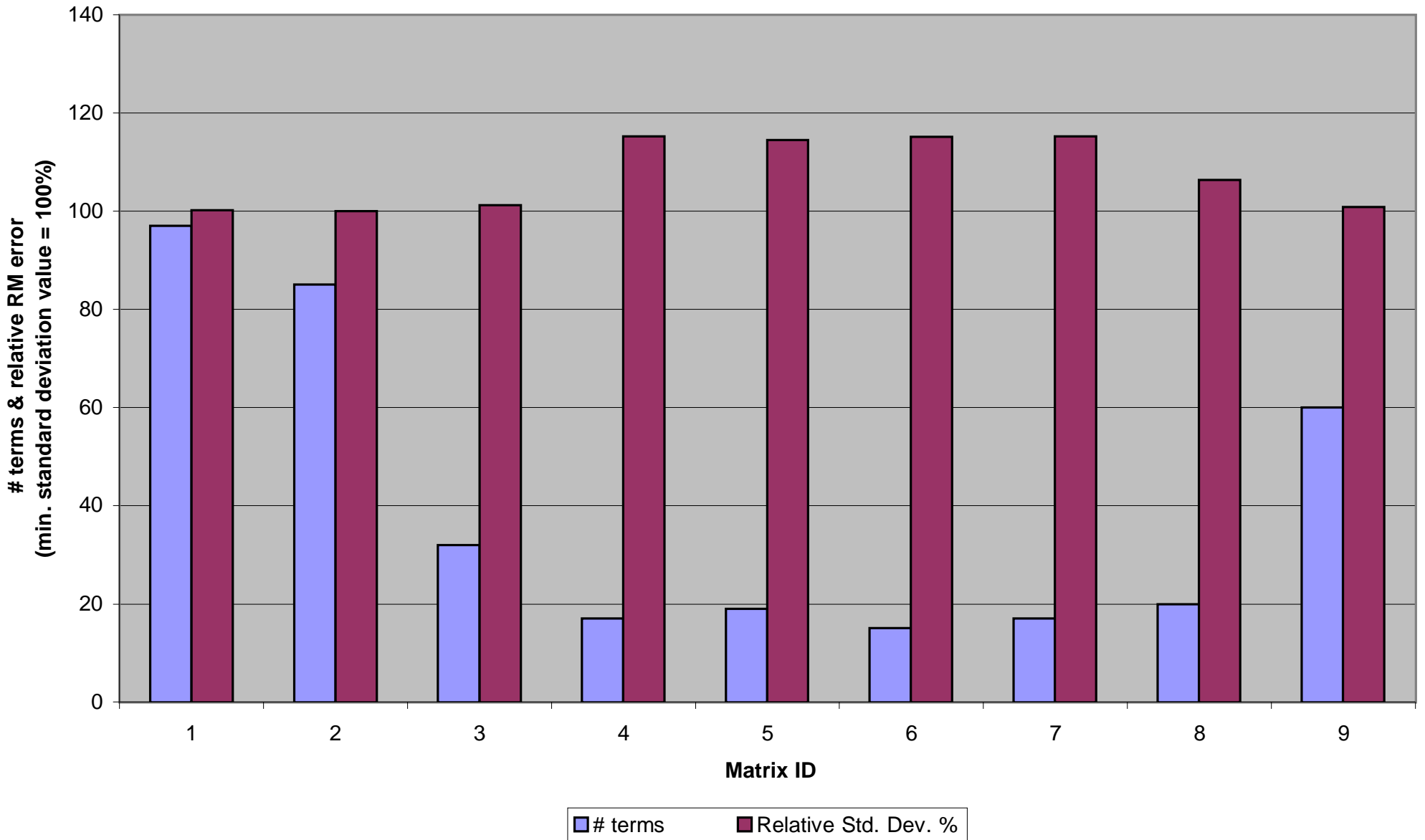
**Mk.2A balance, Y1 error relative standard deviations for
Calibration Loads (File 07)**



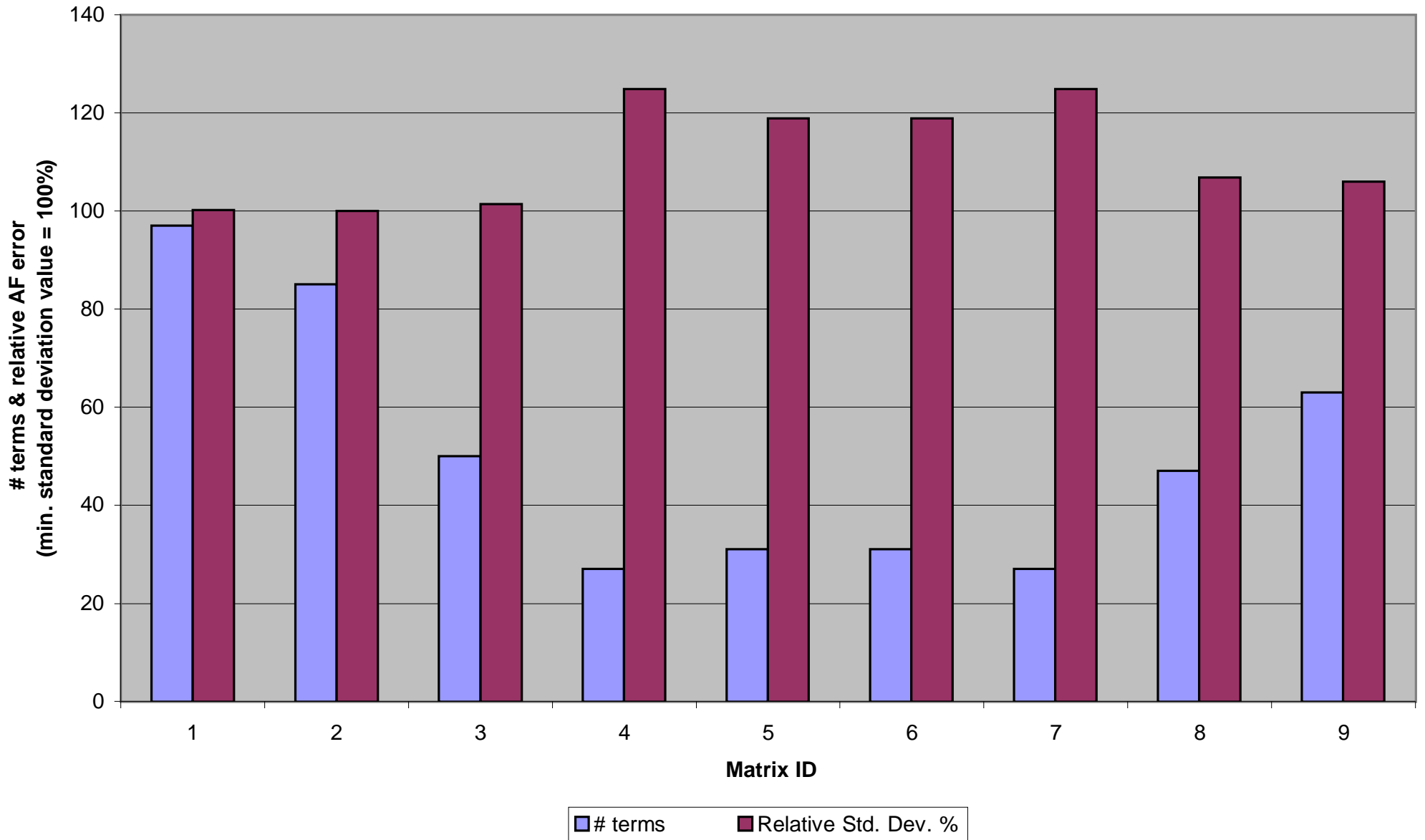
**Mk.2A balance, Y2 error relative standard deviations for
Calibration Loads (File 07)**



**Mk.2A balance, RM error relative standard deviations for
Calibration Loads (File 07)**



Mk.2A balance, AF error relative standard deviations for
Calibration Loads (File 07)



Error statistics summary from loads calculation of the Mk.2A Manual Loads File 06 using different matrices.

| Matrix "1" (6x97) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.182 | 0.756 | 1.045 | 1.674 | 2.208 | 0.322 |
| MIN error %FS | -0.284 | -0.890 | -0.287 | -2.473 | -2.603 | -0.239 |
| Absolute largest error %FS | 0.284 | 0.890 | 1.045 | 2.473 | 2.603 | 0.322 |
| Std. Devn. %FS | 0.079 | 0.330 | 0.254 | 0.910 | 0.736 | 0.124 |
| Average error %FS | -0.027 | -0.203 | 0.188 | 0.172 | -0.226 | 0.002 |
| Ratio of absolute largest error to Std. Devn. | 3.62 | 2.70 | 4.11 | 2.72 | 3.54 | 2.60 |
| # terms | 97 | 97 | 97 | 97 | 97 | 97 |

| Matrix "2" (6x85) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.177 | 0.807 | 1.023 | 1.086 | 1.843 | 0.359 |
| MIN error %FS | -0.282 | -0.741 | -0.415 | -2.663 | -1.761 | -0.215 |
| Absolute largest error %FS | 0.282 | 0.807 | 1.023 | 2.663 | 1.843 | 0.359 |
| Std. Devn. %FS | 0.073 | 0.306 | 0.247 | 0.855 | 0.612 | 0.117 |
| Average error %FS | -0.012 | 0.012 | 0.144 | -0.293 | 0.088 | 0.018 |
| Ratio of absolute largest error to Std. Devn. | 3.88 | 2.63 | 4.14 | 3.12 | 3.01 | 3.08 |
| # terms | 85 | 85 | 85 | 85 | 85 | 85 |

| Matrix "3" (Norbert Ulbrich "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.188 | 0.908 | 0.959 | 1.749 | 1.817 | 0.392 |
| MIN error %FS | -0.260 | -0.597 | -0.367 | -1.835 | -2.010 | -0.216 |
| Absolute largest error %FS | 0.260 | 0.908 | 0.959 | 1.835 | 2.010 | 0.392 |
| Std. Devn. %FS | 0.068 | 0.311 | 0.231 | 0.736 | 0.612 | 0.117 |
| Average error %FS | -0.004 | 0.042 | 0.085 | -0.040 | 0.062 | 0.023 |
| Ratio of absolute largest error to Std. Devn. | 3.80 | 2.92 | 4.14 | 2.49 | 3.29 | 3.34 |
| # terms | 52 | 42 | 38 | 37 | 32 | 50 |

| Matrix "4" (Mod-A of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.205 | 0.828 | 0.969 | 2.151 | 1.399 | 0.315 |
| MIN error %FS | -0.275 | -0.733 | -0.512 | -1.983 | -2.291 | -0.301 |
| Absolute largest error %FS | 0.275 | 0.828 | 0.969 | 2.151 | 2.291 | 0.315 |
| Std. Devn. %FS | 0.062 | 0.304 | 0.242 | 0.810 | 0.632 | 0.109 |
| Average error %FS | -0.007 | 0.017 | 0.068 | -0.070 | -0.043 | -0.004 |
| Ratio of absolute largest error to Std. Devn. | 4.42 | 2.72 | 4.01 | 2.66 | 3.63 | 2.89 |
| # terms | 32 | 28 | 26 | 14 | 17 | 27 |

| Matrix "5" (Mod-A2 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.179 | 0.858 | 0.984 | 1.478 | 1.259 | 0.347 |
| MIN error %FS | -0.271 | -0.740 | -0.483 | -2.304 | -2.185 | -0.249 |
| Absolute largest error %FS | 0.271 | 0.858 | 0.984 | 2.304 | 2.185 | 0.347 |
| Std. Devn. %FS | 0.059 | 0.293 | 0.237 | 0.789 | 0.593 | 0.110 |
| Average error %FS | -0.015 | 0.010 | 0.105 | -0.174 | -0.025 | 0.007 |
| Ratio of absolute largest error to Std. Devn. | 4.56 | 2.92 | 4.16 | 2.92 | 3.68 | 3.15 |
| # terms | 38 | 26 | 32 | 24 | 19 | 31 |

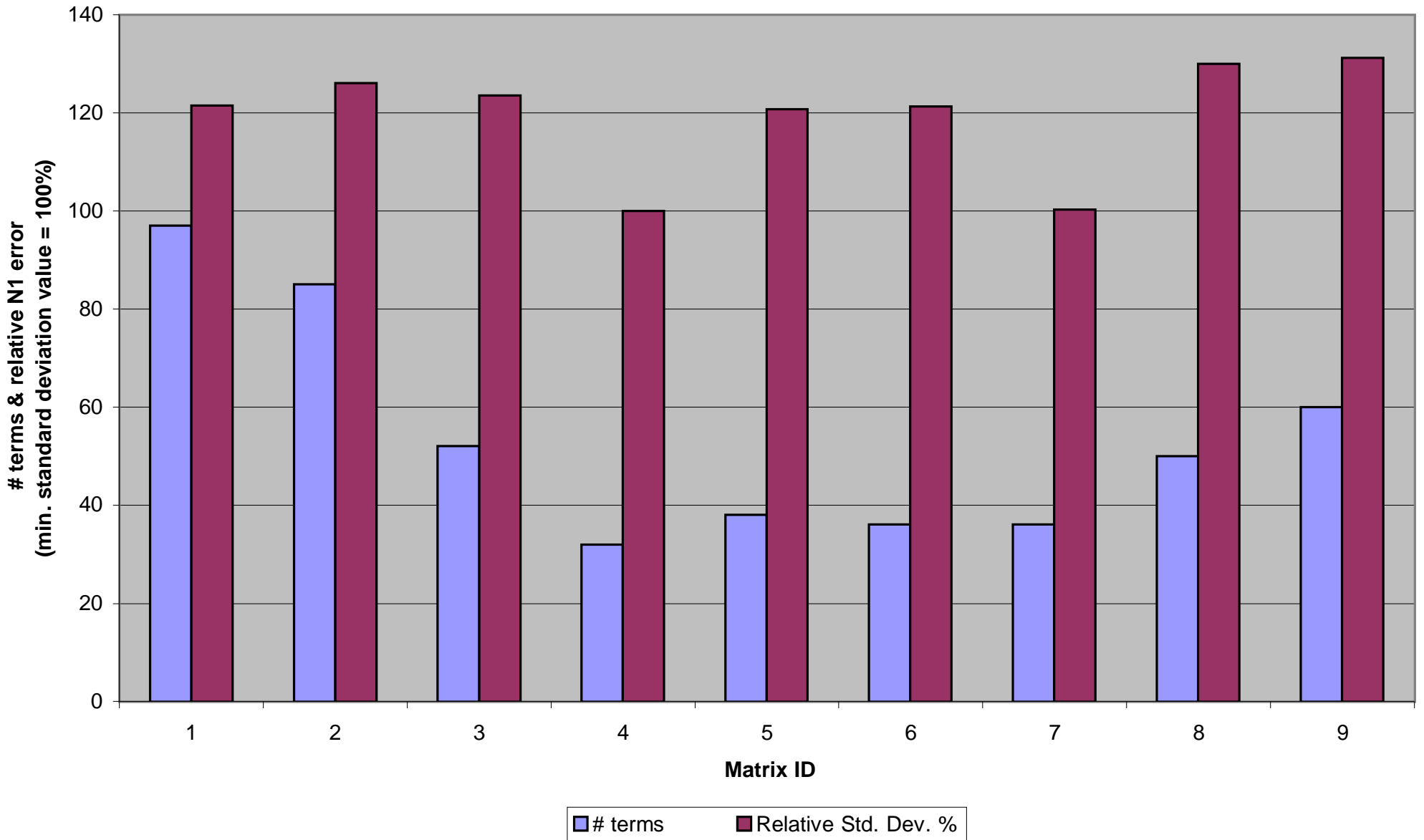
| Matrix "6" (Mod-A3 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.178 | 0.855 | 0.978 | 1.940 | 1.240 | 0.347 |
| MIN error %FS | -0.270 | -0.738 | -0.502 | -2.051 | -2.219 | -0.249 |
| Absolute largest error %FS | 0.270 | 0.855 | 0.978 | 2.051 | 2.219 | 0.347 |
| Std. Devn. %FS | 0.059 | 0.293 | 0.235 | 0.789 | 0.596 | 0.110 |
| Average error %FS | -0.014 | 0.009 | 0.072 | -0.056 | -0.072 | 0.006 |
| Ratio of absolute largest error to Std. Devn. | 4.55 | 2.92 | 4.16 | 2.60 | 3.73 | 3.15 |
| # terms | 36 | 26 | 30 | 20 | 15 | 31 |

| Matrix "7" (Mod-B of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.206 | 0.829 | 0.969 | 2.166 | 1.399 | 0.314 |
| MIN error %FS | -0.285 | -0.732 | -0.532 | -1.985 | -2.291 | -0.302 |
| Absolute largest error %FS | 0.285 | 0.829 | 0.969 | 2.166 | 2.291 | 0.314 |
| Std. Devn. %FS | 0.064 | 0.304 | 0.243 | 0.808 | 0.632 | 0.109 |
| Average error %FS | -0.007 | 0.017 | 0.069 | -0.054 | -0.043 | -0.004 |
| Ratio of absolute largest error to Std. Devn. | 4.45 | 2.72 | 3.98 | 2.68 | 3.63 | 2.88 |
| # terms | 36 | 28 | 26 | 22 | 17 | 27 |

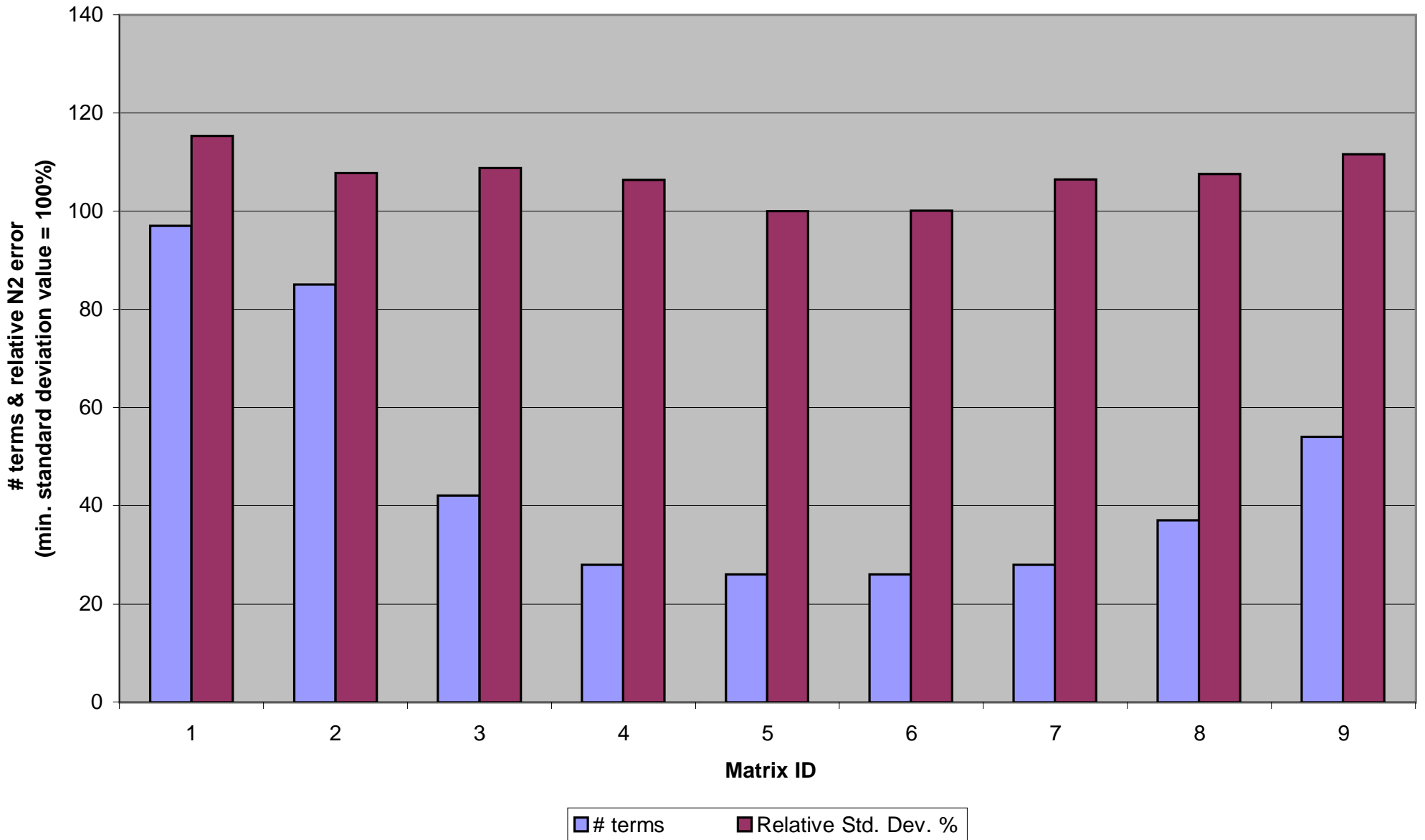
| Matrix "8" (Mod-C of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.202 | 0.864 | 0.971 | 2.087 | 1.491 | 0.373 |
| MIN error %FS | -0.282 | -0.656 | -0.362 | -1.994 | -2.080 | -0.217 |
| Absolute largest error %FS | 0.282 | 0.864 | 0.971 | 2.087 | 2.080 | 0.373 |
| Std. Devn. %FS | 0.072 | 0.305 | 0.246 | 0.806 | 0.605 | 0.114 |
| Average error %FS | -0.003 | 0.027 | 0.087 | -0.066 | -0.009 | 0.018 |
| Ratio of absolute largest error to Std. Devn. | 3.92 | 2.83 | 3.95 | 2.59 | 3.44 | 3.28 |
| # terms | 50 | 37 | 32 | 33 | 20 | 47 |

| Matrix "9" (Mod-D of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.199 | 0.903 | 0.944 | 2.162 | 1.865 | 0.375 |
| MIN error %FS | -0.285 | -0.610 | -0.351 | -2.043 | -2.020 | -0.226 |
| Absolute largest error %FS | 0.285 | 0.903 | 0.944 | 2.162 | 2.020 | 0.375 |
| Std. Devn. %FS | 0.071 | 0.311 | 0.240 | 0.798 | 0.628 | 0.114 |
| Average error %FS | -0.004 | 0.033 | 0.086 | -0.070 | 0.062 | 0.019 |
| Ratio of absolute largest error to Std. Devn. | 4.01 | 2.90 | 3.93 | 2.71 | 3.22 | 3.28 |
| # terms | 60 | 54 | 50 | 54 | 60 | 63 |

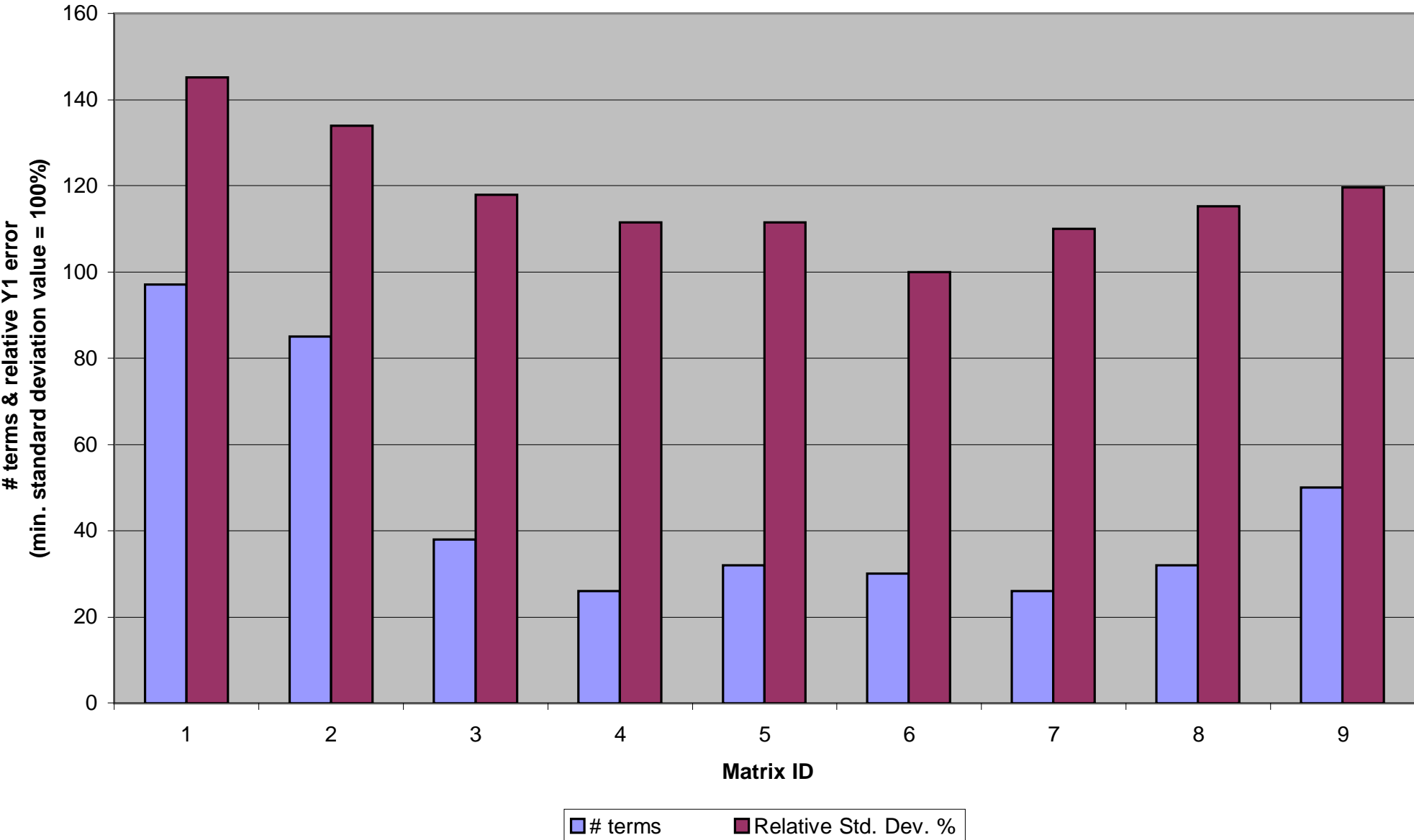
**Mk.2A balance, N1 error relative standard deviations for
Manual Loads (File 06)**



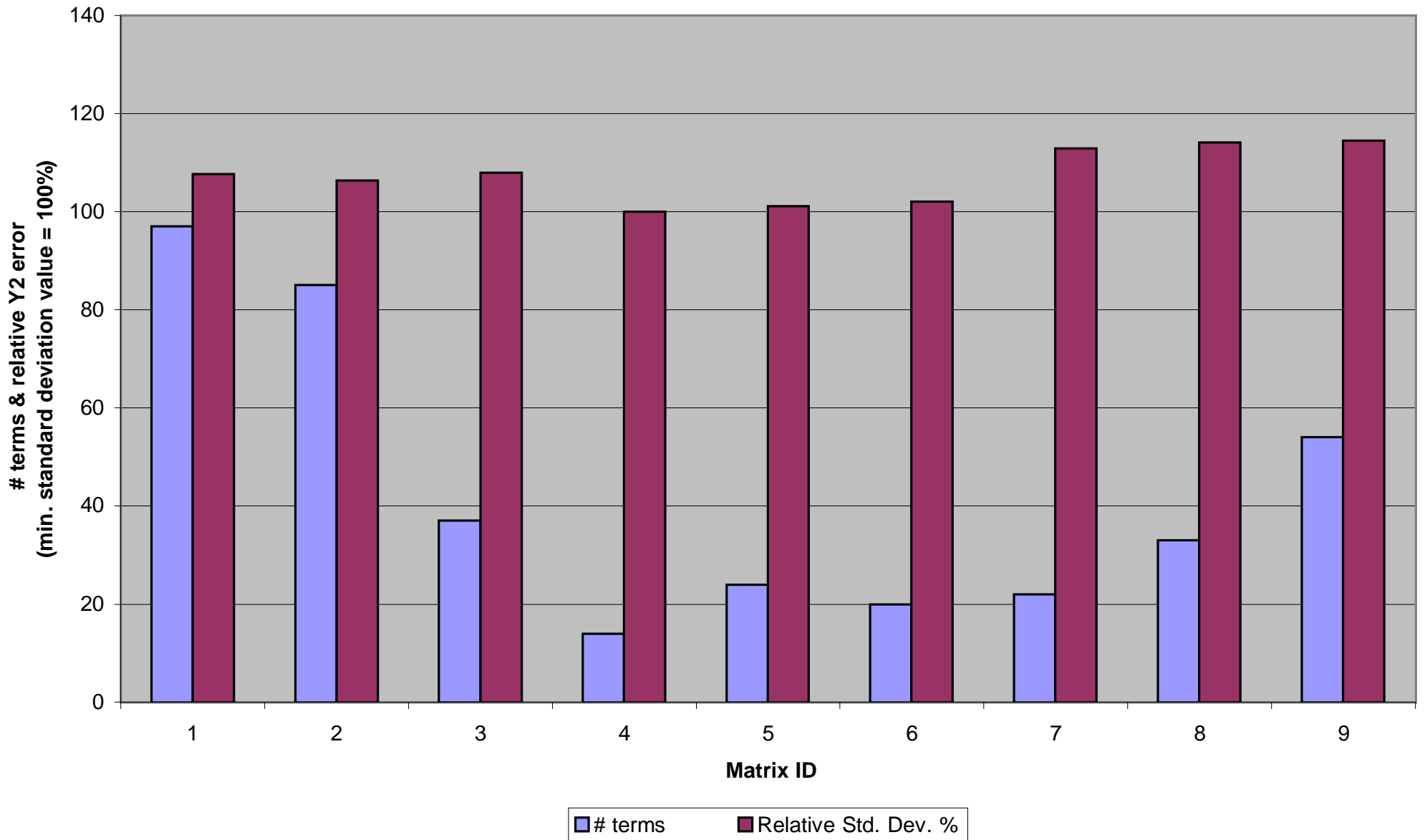
**Mk.2A balance, N2 error relative standard deviations for
Manual Loads (File 06)**



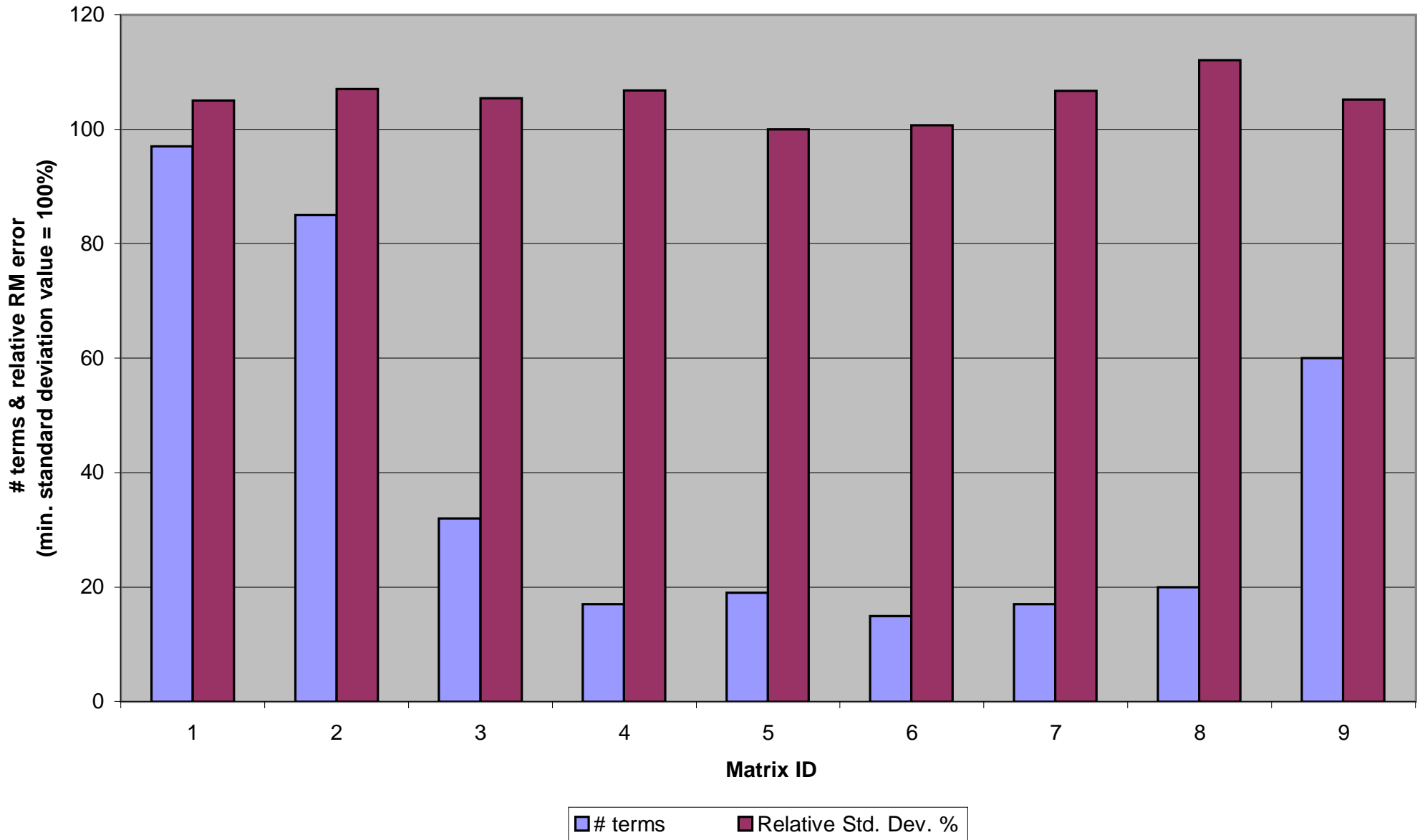
Mk.2A balance, Y1 error relative standard deviations for
Manual Loads (File 06)



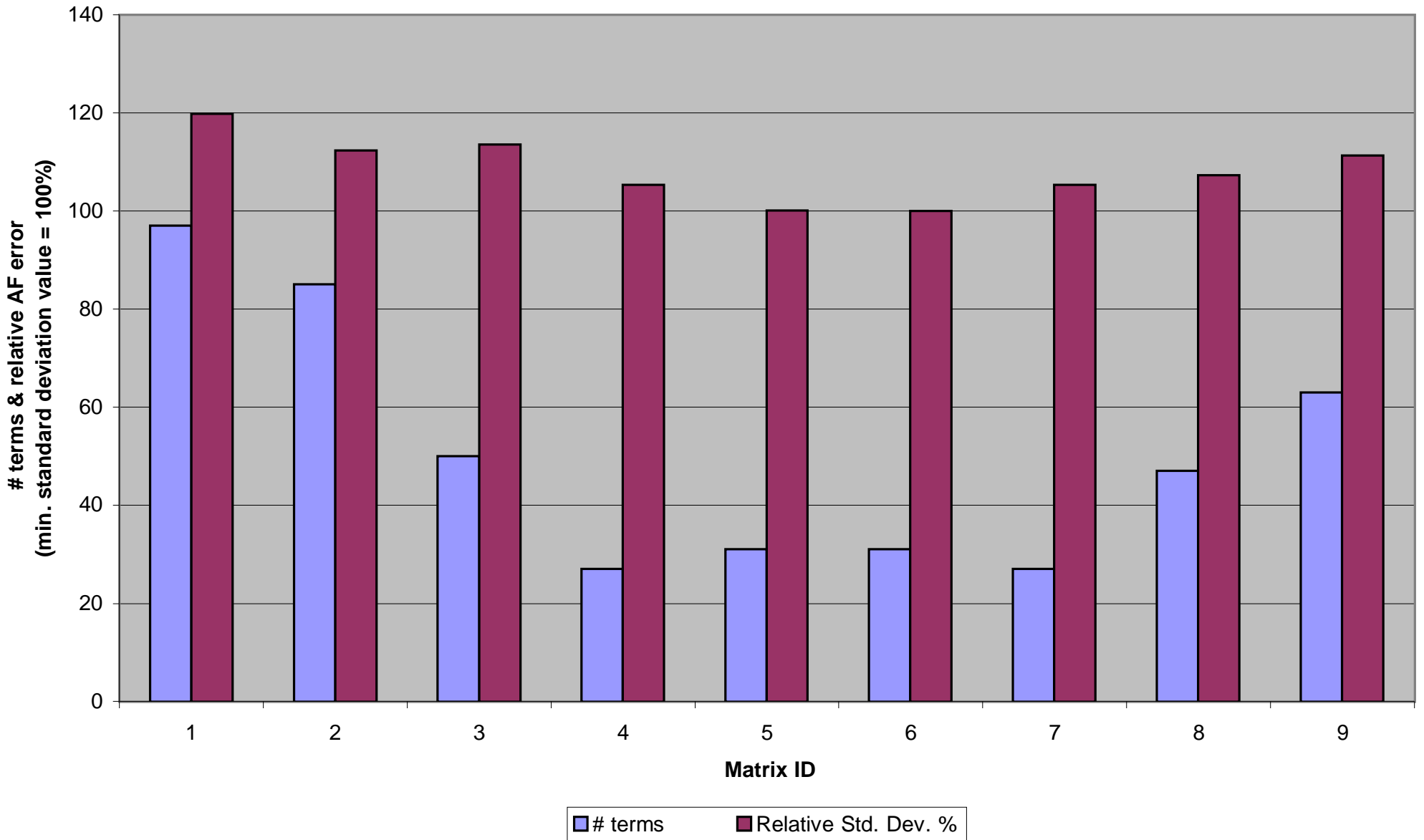
**Mk.2A balance, Y2 error relative standard deviations for
Manual Loads (File 06)**



**Mk.2A balance, RM error relative standard deviations for
Manual Loads (File 06)**



Mk.2A balance, AF error relative standard deviations for
Manual Loads (File 06)



Error statistics summary from loads calculation of the Mk.2A generic 6-component W/T Test Loads File 03 using different matrices.

| Matrix "1" (6x97) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.022 | 0.913 | 0.969 | 2.228 | 2.867 | 0.317 |
| MIN error %FS | -0.478 | -1.161 | -0.192 | -3.821 | -2.998 | -0.412 |
| Absolute largest error %FS | 0.478 | 1.161 | 0.969 | 3.821 | 2.998 | 0.412 |
| Std. Devn. %FS | 0.104 | 0.467 | 0.226 | 1.110 | 1.205 | 0.167 |
| Average error %FS | -0.256 | -0.127 | 0.191 | -0.571 | -0.245 | -0.051 |
| Ratio of absolute largest error to Std. Devn. | 4.60 | 2.48 | 4.28 | 3.44 | 2.49 | 2.46 |
| # terms | 97 | 97 | 97 | 97 | 97 | 97 |

| Matrix "2" (6x85) | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.088 | 0.636 | 0.849 | 2.411 | 2.435 | 0.220 |
| MIN error %FS | -0.477 | -1.086 | -0.291 | -3.427 | -3.280 | -0.418 |
| Absolute largest error %FS | 0.477 | 1.086 | 0.849 | 3.427 | 3.280 | 0.418 |
| Std. Devn. %FS | 0.108 | 0.437 | 0.209 | 1.097 | 1.228 | 0.157 |
| Average error %FS | -0.233 | -0.262 | 0.099 | -0.360 | -0.760 | -0.081 |
| Ratio of absolute largest error to Std. Devn. | 4.43 | 2.49 | 4.07 | 3.12 | 2.67 | 2.67 |
| # terms | 85 | 85 | 85 | 85 | 85 | 85 |

| Matrix "3" (Norbert Ulbrich "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.099 | 0.673 | 0.512 | 2.999 | 2.384 | 0.228 |
| MIN error %FS | -0.450 | -1.062 | -0.570 | -2.176 | -3.290 | -0.415 |
| Absolute largest error %FS | 0.450 | 1.062 | 0.570 | 2.999 | 3.290 | 0.415 |
| Std. Devn. %FS | 0.106 | 0.441 | 0.184 | 1.114 | 1.210 | 0.158 |
| Average error %FS | -0.218 | -0.260 | -0.075 | 0.506 | -0.763 | -0.075 |
| Ratio of absolute largest error to Std. Devn. | 4.26 | 2.41 | 3.10 | 2.69 | 2.72 | 2.62 |
| # terms | 52 | 42 | 38 | 37 | 32 | 50 |

| Matrix "4" (Mod-A of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.146 | 0.633 | 0.284 | 2.669 | 2.363 | 0.180 |
| MIN error %FS | -0.365 | -1.102 | -0.742 | -2.240 | -3.427 | -0.484 |
| Absolute largest error %FS | 0.365 | 1.102 | 0.742 | 2.669 | 3.427 | 0.484 |
| Std. Devn. %FS | 0.085 | 0.431 | 0.174 | 1.032 | 1.226 | 0.147 |
| Average error %FS | -0.203 | -0.279 | -0.188 | 0.410 | -0.966 | -0.176 |
| Ratio of absolute largest error to Std. Devn. | 4.27 | 2.55 | 4.27 | 2.59 | 2.80 | 3.30 |
| # terms | 32 | 28 | 26 | 14 | 17 | 27 |

| Matrix "5" (Mod-A2 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.073 | 0.562 | 0.594 | 2.431 | 1.878 | 0.229 |
| MIN error %FS | -0.432 | -1.103 | -0.380 | -2.573 | -3.466 | -0.398 |
| Absolute largest error %FS | 0.432 | 1.103 | 0.594 | 2.573 | 3.466 | 0.398 |
| Std. Devn. %FS | 0.103 | 0.405 | 0.174 | 1.043 | 1.148 | 0.140 |
| Average error %FS | -0.242 | -0.305 | -0.003 | 0.148 | -1.108 | -0.107 |
| Ratio of absolute largest error to Std. Devn. | 4.18 | 2.72 | 3.42 | 2.47 | 3.02 | 2.85 |
| # terms | 38 | 26 | 32 | 24 | 19 | 31 |

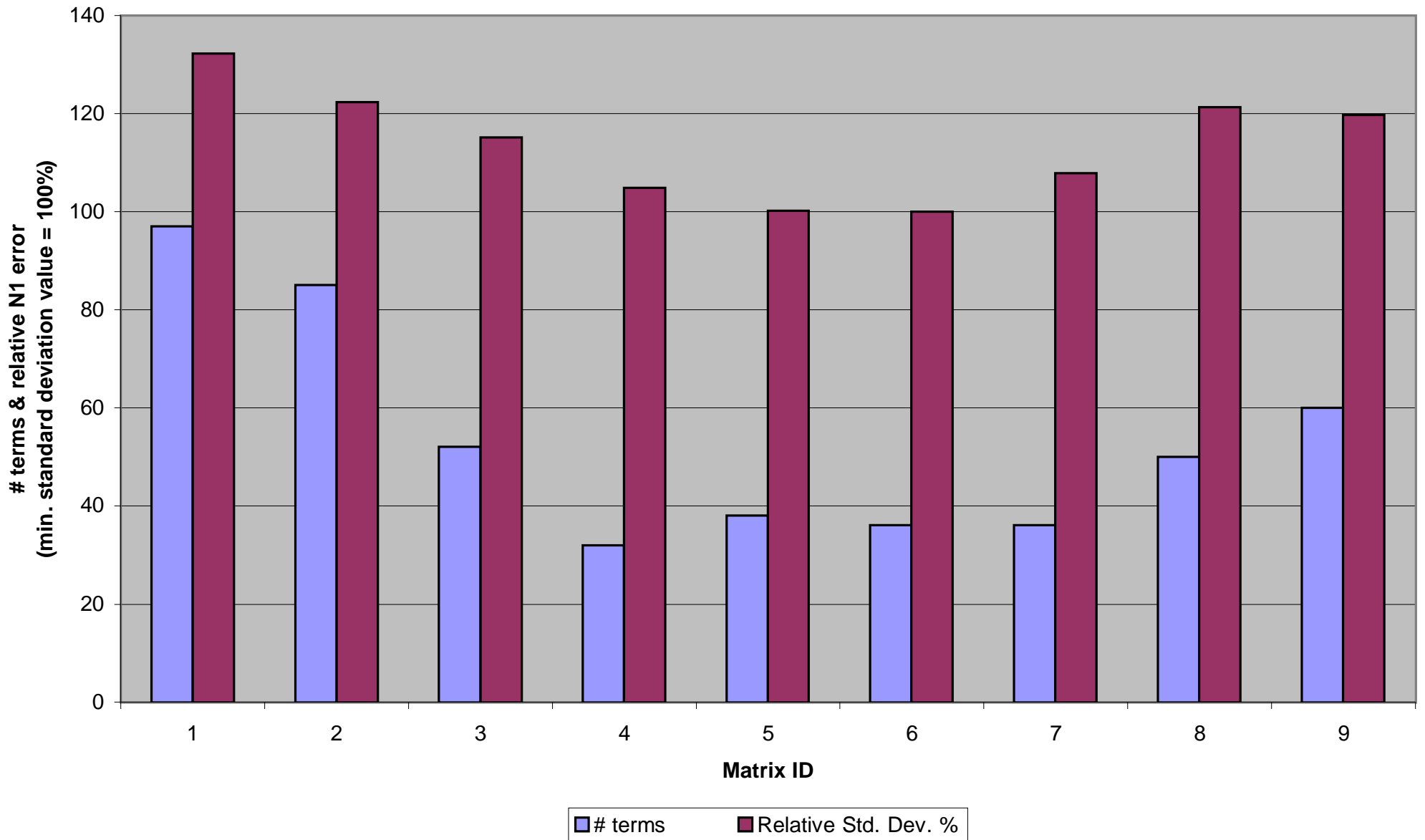
| Matrix "6" (Mod-A3 of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.073 | 0.562 | 0.337 | 2.855 | 1.733 | 0.228 |
| MIN error %FS | -0.431 | -1.104 | -0.567 | -1.972 | -3.610 | -0.399 |
| Absolute largest error %FS | 0.431 | 1.104 | 0.567 | 2.855 | 3.610 | 0.399 |
| Std. Devn. %FS | 0.104 | 0.406 | 0.156 | 1.053 | 1.156 | 0.139 |
| Average error %FS | -0.240 | -0.305 | -0.113 | 0.568 | -1.225 | -0.108 |
| Ratio of absolute largest error to Std. Devn. | 4.16 | 2.72 | 3.64 | 2.71 | 3.12 | 2.86 |
| # terms | 36 | 26 | 30 | 20 | 15 | 31 |

| Matrix "7" (Mod-B of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.135 | 0.635 | 0.297 | 2.602 | 2.359 | 0.180 |
| MIN error %FS | -0.371 | -1.102 | -0.738 | -2.436 | -3.429 | -0.484 |
| Absolute largest error %FS | 0.371 | 1.102 | 0.738 | 2.602 | 3.429 | 0.484 |
| Std. Devn. %FS | 0.086 | 0.432 | 0.171 | 1.165 | 1.225 | 0.147 |
| Average error %FS | -0.207 | -0.279 | -0.187 | 0.436 | -0.967 | -0.176 |
| Ratio of absolute largest error to Std. Devn. | 4.33 | 2.55 | 4.31 | 2.23 | 2.80 | 3.29 |
| # terms | 36 | 28 | 26 | 22 | 17 | 27 |

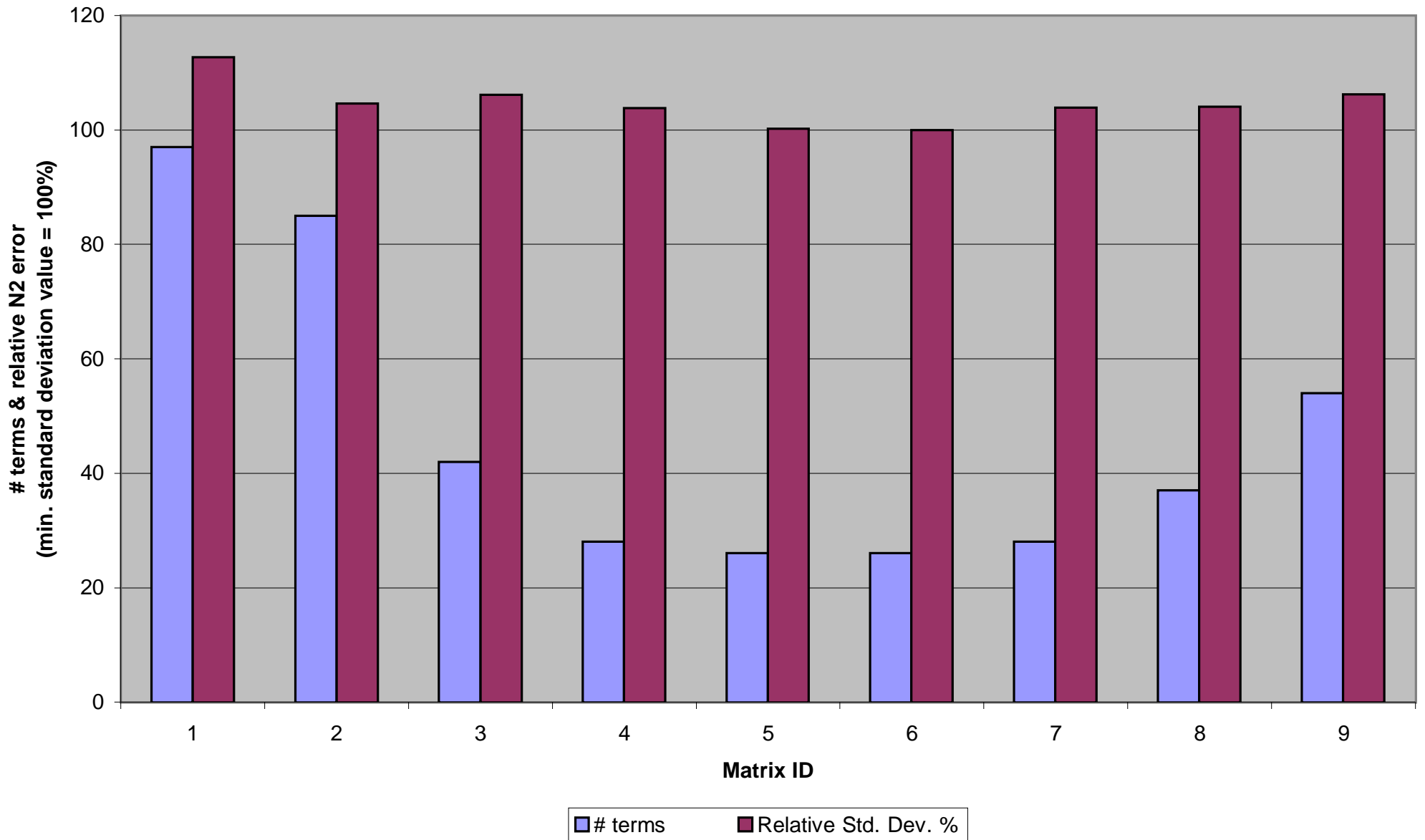
| Matrix "8" (Mod-C of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.129 | 0.676 | 0.370 | 3.080 | 2.462 | 0.248 |
| MIN error %FS | -0.468 | -1.075 | -0.689 | -2.539 | -3.605 | -0.417 |
| Absolute largest error %FS | 0.468 | 1.075 | 0.689 | 3.080 | 3.605 | 0.417 |
| Std. Devn. %FS | 0.111 | 0.436 | 0.180 | 1.178 | 1.286 | 0.150 |
| Average error %FS | -0.221 | -0.246 | -0.135 | 0.524 | -1.021 | -0.107 |
| Ratio of absolute largest error to Std. Devn. | 4.21 | 2.47 | 3.84 | 2.61 | 2.80 | 2.79 |
| # terms | 50 | 37 | 32 | 33 | 20 | 47 |

| Matrix "9" (Mod-D of "optimum") | N1 (%FS) | N2 (%FS) | Y1 (%FS) | Y2 (%FS) | RM (%FS) | AF (%FS) |
|---|----------|----------|----------|----------|----------|----------|
| MAX error %FS | 0.131 | 0.706 | 0.441 | 2.932 | 2.196 | 0.230 |
| MIN error %FS | -0.473 | -1.062 | -0.687 | -2.588 | -3.299 | -0.422 |
| Absolute largest error %FS | 0.473 | 1.062 | 0.687 | 2.932 | 3.299 | 0.422 |
| Std. Devn. %FS | 0.112 | 0.452 | 0.186 | 1.181 | 1.207 | 0.155 |
| Average error %FS | -0.221 | -0.251 | -0.112 | 0.450 | -0.866 | -0.115 |
| Ratio of absolute largest error to Std. Devn. | 4.22 | 2.35 | 3.68 | 2.48 | 2.73 | 2.72 |
| # terms | 60 | 54 | 50 | 54 | 60 | 63 |

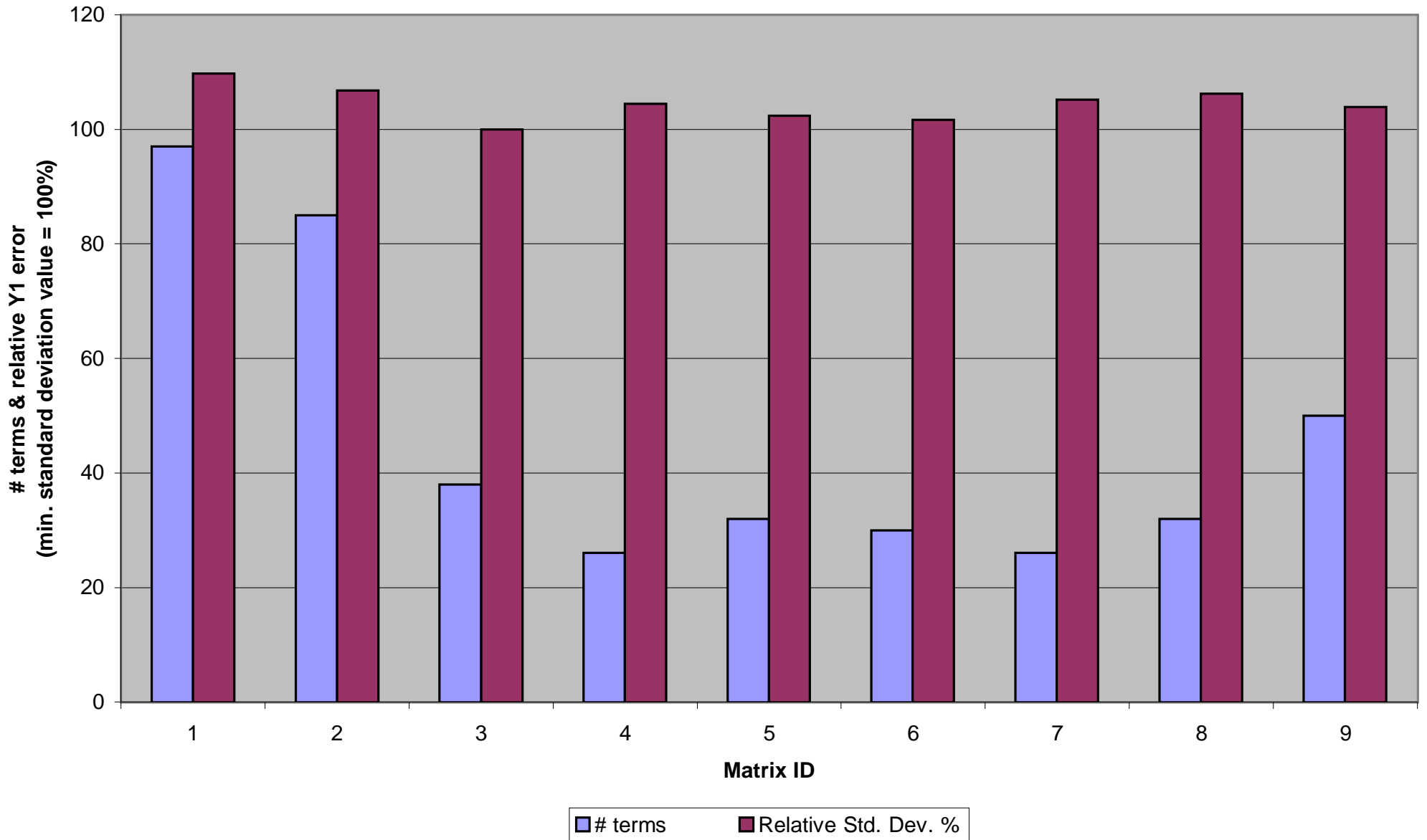
**Mk.2A balance, N1 error relative standard deviations for
6-component generic W/T Test Loads (File 03)**



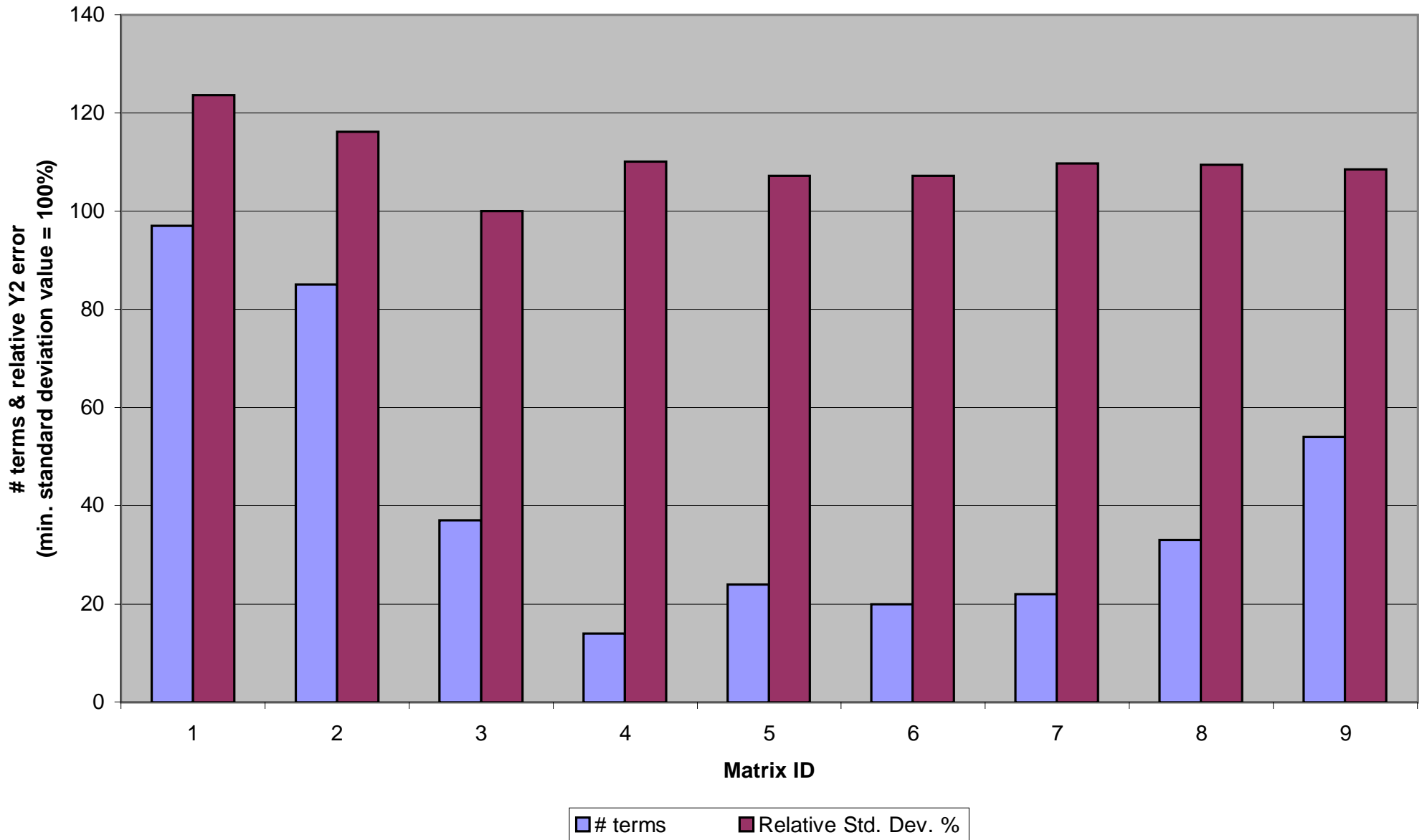
**Mk.2A balance, N2 error relative standard deviations for
6-component generic W/T Test Loads (File 03)**



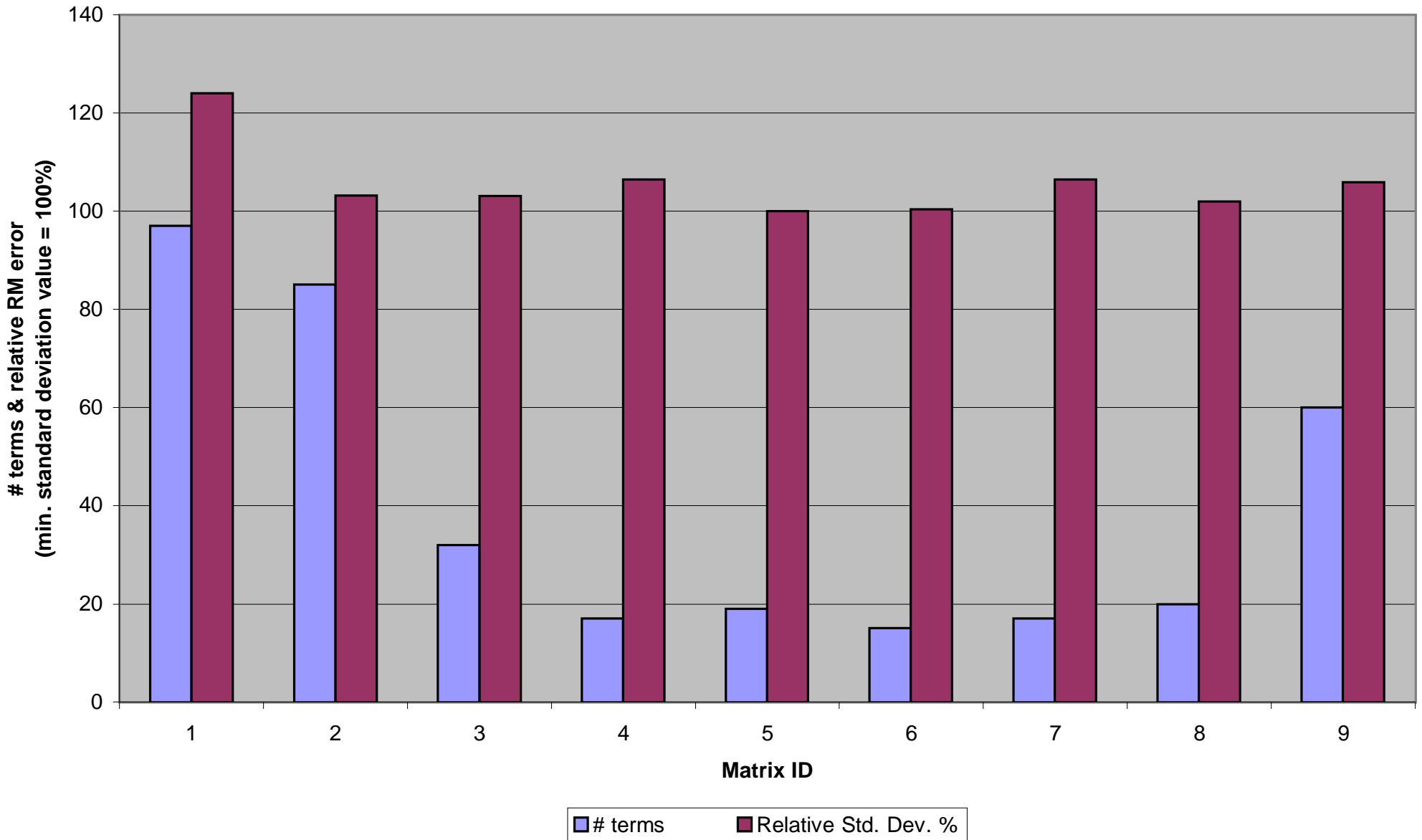
**Mk.2A balance, Y1 error relative standard deviations for
6-component generic W/T Test Loads (File 03)**



**Mk.2A balance, Y2 error relative standard deviations for
6-component generic W/T Test Loads (File 03)**



**Mk.2A balance, RM error relative standard deviations for
6-component generic W/T Test Loads (File 03)**



**Mk.2A balance, AF error relative standard deviations for
6-component generic W/T Test Loads (File 03)**

