The background history of the AIAA 6x96 balance math model

Early on in the AIAA Balance Working Group discussions, the members agreed that the math model best describing the load-response relationship for strain gauge balances was that given by Equation 3.1.2 of AIAA R-091-2003, namely:

$$R_i = a_i + \sum_{j=1}^{n} b_{ij} F_j + \sum_{j=1}^{n} \sum_{k=j}^{n} c_{i,j,k} F_j F_k + \sum_{j=1}^{n} d_{ij} F_j^3$$

Several members indicated that their calibration procedures often determined two (or four) different values of the coefficients, to be used with positive and negative loads (or load product sign combinations) so as to model asymmetric load behaviour. The coefficients of the various load terms typically were determined by a process of 'piecewise curve-fitting', as mentioned in AIAA R-091-2003. In that process the coefficients for a particular sign of load, (or sign combination of a load product), were derived from just a subset of calibration data for that particular load sign or sign combination. This procedure easily allowed for calibrations in which some coefficients had multiple values to account for asymmetric load effects, while such effects were ignored for other coefficients that thus had only a single value. The decision of whether to derive 'sign-dependent' or 'sign-independent' coefficients usually would be based on the characteristics of the particular balance, and/or the load combinations that were likely to be encountered in its use. Sign-dependent coefficients of the F_j, F_j^2, and F_j^3 terms could assume one of 2 values depending on the sign of F_j, while the coefficients of the $F_j.F_k$ terms could assume one of 4 values depending on the sign combination of the loads in the product.

When the working group discussion turned to the use of global regression as being the only reasonable technique for processing the type of calibration dataset typically acquired with automatic calibration machines, most of the group members who were accustomed to accounting for asymmetric load behaviour in the above manner, expressed concern that they would not be able to do so if they used global regression. This was the point at which I proposed the 'signed and absolute value of load' technique as a way of allowing the use of global regression while retaining the ability to account for asymmetric load behaviour. This was a technique that I had encountered in the early 1970's, when I first came across it being applied to just the linear terms in a matrix to account for linear change of slope through zero load. At that time I extended the technique to the higher order terms, and have used it ever since then.

That point in the balance working group discussions really marks the 'birth' of what has come to be called the *AIAA 6x96 recommended math model*! For a 6-component balance, the model of Eq. 3.1.2 defines a 6x33 matrix, (6x34 with the intercept), and this model ignores any asymmetric load behaviour. The *AIAA 6x96 model* is just the consequence of applying asymmetric load behaviour to that 6x33 model, nothing more, nothing less! That 6x96 model, (6x97 with intercept), was never intended (in my mind at least) to be "*the model* " that everyone should use, but rather was presented as an alternative to the model defined by Eq. 3.1.2 which allowed for load asymmetry effects when the data were analysed using global regression. The AIAA Recommended Practice document includes the following statement concerning the recommended 6x96 model:

> *"However, it is stressed that this recommendation does not imply that all of the terms in the full math model must be defined, (or even should be defined), in the calibration of any given balance. It is good engineering practice to design the balance calibration to meet the objectives of the intended test, and if these objectives can be realized using a less complex calibration, (which omits selected terms from the full model), then additional calibration effort (and cost) is not justified."*

However, despite that statement it seems that the full 96-term (plus intercept) model is all that people now think of when the AIAA recommended math model is mentioned! I regard that as being rather unfortunate, but that's the way it is! This model is represented by Eq. 3.1.3 in AIAA RP-091-2003:

$$R_i = a_i + \sum_{j=1}^{n} b1_{ij}\, F_j + \sum_{j=1}^{n} b2_{ij} \left| F_j \right| +$$

$$\sum_{j=1}^{n} c1_{ij}\, F_j^2 + \sum_{j=1}^{n} c2_{ij}\, F_j \cdot \left| F_j \right| +$$

$$\sum_{j=1}^{n} \sum_{k=j+1}^{n} c3_{ij,k}\, F_j \cdot F_k + \sum_{j=1}^{n} \sum_{k=j+1}^{n} c4_{ij,k} \left| F_j \cdot F_k \right| +$$

$$\sum_{j=1}^{n} \sum_{k=j+1}^{n} c5_{ij,k}\, F_j \cdot \left| F_k \right| + \sum_{j=1}^{n} \sum_{k=j+1}^{n} c6_{ij,k} \left| F_j \right| \cdot F_k +$$

$$\sum_{j=1}^{n} d1_{ij}\, F_j^3 + \sum_{j=1}^{n} d2_{ij} \left| F_j^3 \right|$$

A year or so ago Norbert and I had a quite extensive E-mail discussion regarding the way in which the 'optimization' of this *AIAA recommended* model had been implemented in BALFIT, since our views of what constituted the ‘basic’ model differed. I regarded Eq. 3.1.2 as defining the ‘basic’ model with **four** ‘math term groups’, while Norbert considered the ‘basic’ model to be that defined by Eq. 3.1.3, which he regarded as having **ten** ‘math term groups’, all independent of each other.

Since I introduced this model at the working group I know exactly how I intended it to be used, and that was simply as an emulation of the model of Eq. 3.1.2. Thus I regard the model of Eq. 3.1.3 as having just **four** ‘unrelated’ math term groups. Three of these groups, (F_j, F_j^2, and F_j^3), each contain pairs of ‘related’ terms corresponding to the b1, b2, c1, c2, d1 and d2 coefficients in Eq. 3.1.3, and the forth group, ($F_j.F_k$), contains a quadruplet of ‘related’ terms corresponding to the c3, c4, c5, and c6 coefficients. *My position is that, within each of the four basic groups, the terms involving 'signed' and 'absolute value' of load are inter-related, and that either all of the terms (2 or 4 as the case may be) should be used together, or the 'signed' term should be used on its own.* My reason for proposing this constraint is that the related terms in any group in Eq. 3.1.3 can be derived from the multiple values of the coefficient for the corresponding term group in Eq. 3.1.2, only if **all** of the 2 or 4 multiple values are defined.

Attached is a simple Excel spreadsheet that demonstrates the relationships between the coefficients in the two equations. The values shown in the cells with red font are selected to show easily recognizable differences in the resulting value of the contribution from each ‘term group’ to the bridge response, as the signs of F_j and F_k are varied. (The values in any of the cells shown in red font may be changed; the formulae cells are locked.) Simulating an ‘undefined’ coefficient, e.g. by entering a non-numeric value for any coefficient, results in an error.

I proposed to Norbert that there should be additional constraints imposed in the optimization process that would enforce the condition *outlined above*, and described the constraints I thought appropriate as follows:

If, in the 97-term math model equation,

1) *the coefficient of a 'signed' term for any strain gauge bridge in the groups (F_j, F_j^2, and F_j^3 for j=1,6) is SET TO ZERO (by BALFIT's algorithm), then the coefficient of the corresponding 'absolute value' term should also be SET TO ZERO regardless of whether or not BALFIT's algorithm would zero it.*

2) *the coefficient of a 'signed' term for any strain gauge bridge in the group ($F_j.F_k$ for j=1,6, k=J+1,6) is SET TO ZERO (by BALFIT's algorithm), then the coefficients of the corresponding THREE 'absolute value' terms should be SET TO ZERO regardless of whether or not BALFIT's algorithm would zero any or all of them.*

3) *the coefficient of a 'signed' term for any strain gauge bridge in the group ($F_j.F_k$ for j=1,6, k=J+1,6) is NON-ZERO, (i.e. is selected by BALFIT's algorithm), but ANY of the corresponding THREE 'absolute value' terms are SET TO ZERO (by BALFIT's algorithm), then ALL THREE of those 'absolute value' terms should be SET TO ZERO regardless of whether or not BALFIT's algorithm would zero any or all of them.*

Of course no term that BALFIT would exclude is actually 'undefined' mathematically, it's just assigned a value of zero. Thus the coefficients of the two equations can always be transformed, one to the other. But I see the "multi-valued" coefficients in Eq. 3.1.2 as having a logical, easily recognized, relationship to the physical reality of how the balance behaves, unlike the "invariant" coefficients in Eq. 3.1.3. When no distinction is made between the effects of positive and negative loads, that just means that all values of the "multi-valued" coefficient for a particular term group are equal, and this translates to zero values for all coefficients in Eq. 3.1.3 <u>except</u> those for the 'signed' term group. When accounting for different behaviour for positive and negative loads (load asymmetry), the two possible values for the coefficients of the F_j, F_j^2, and F_j^3 term groups may be different, while for coefficients of the $F_j.F_k$ term group one could consider the following possibilities for the four values. (I really think that only the first of these makes physical sense, but at least it's possible to 'visualize' how the other options relate to the physical environment.)

1) all four values are potentially different, corresponding to the balance's behaviour in each of the four quadrants defining ++, - -, + -, and -+ sign combinations

2) the four values are associated only with the sign of load F_j, in which case the sign combinations ++ and + - would have the same value, and the combinations - - and -+ could have a different value

3) the four values are associated only with the sign of load F_k, in which case the sign combinations ++ and -+ would have the same value, and the combinations - - and + - could have a different value

4) the four values are associated only with the sign of the load product $F_j * F_k$, in which case the sign combinations ++ and - - would have the same value, and the combinations + - and -+ could have a different value.

Options (2), (3) and (4) each result in two of the four "invariant" coefficients for the $F_j.F_k$ term group having zero values. One or more of these "invariant" coefficients being zero-valued doesn't make the transformation to the "multi-valued" coefficients impossible, but I can't 'visualize' how the $F_j.F_k$, $|F_j.F_k|$, $F_j.|F_k|$, or $|F_j|.F_k$ terms relate to the physical environment in the way that I can for the "multi-valued" coefficients of the terms in Eq. 3.1.2. This is what makes me 'uncomfortable' with the concept of treating those terms as four independent groups, any of which may be retained or excluded without reference to the others. Similarly for the pairs of terms associated with F_j, F_j^2, and F_j^3. That's the reasoning behind my suggestion to impose those additional constraints on BALFIT's optimization algorithm.

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Equation 3.1.2 is the long-accepted basic equation using "multi-valued" coefficients {b_{ij}, $c_{ij.j}$, $c_{ij.k}$, d_{ij}} to model asymmetric load behaviour

LINEAR		LOAD SQUARED		2-LOAD PRODUCT				LOAD CUBED	
b_{ij}		$c_{ij.j}$		$c_{ij.k}$				d_{ij}	
+	-	+	-	+ +	- -	+ -	- +	+	-
1.005	0.995	1.01	0.99	1.1	0.9	0.8	1.05	1.02	0.98

Equation 3.1.3 is not the basic equation! It is just a reformulation of Eq. 3.1.2 using "invariant" coefficients {$b1_{ij}$, $b2_{ij}$, $c1_{ij.j}$, $c2_{ij.j}$, $c3_{ij.k}$, $c4_{ij.k}$, $c5_{ij.k}$, $c6_{ij.k}$, $d1_{ij}$, $d2_{ij}$} in combination with "signed" and "absolute" values of load, to emulate the function of the "multi-valued" coefficients {b_{ij}, $c_{ij.j}$, $c_{ij.k}$, d_{ij}} used in Eq. 3.1.2. These "invariant" coefficients are indeterminate unless ALL (2 or 4) values of the "multi-valued" coefficients are defined.

LINEAR		LOAD SQUARED		2-LOAD PRODUCT				LOAD CUBED	
$b1_{ij}$	$b2_{ij}$	$c1_{ij.j}$	$c2_{ij.j}$	$c3_{ij.k}$	$c4_{ij.k}$	$c5_{ij.k}$	$c6_{ij.k}$	$d1_{ij}$	$d2_{ij}$
F_j	$\|F_j\|$	F_j*F_j	$F_j*\|F_j\|$	F_j*F_k	$\|F_j*F_k\|$	$F_j*\|F_k\|$	$\|F_j\|*F_k$	$F_j*F_j*F_j$	$\|F_j\|*\|F_j\|*\|F_j\|$
1	0.005	1	0.01	0.9625	0.0375	0.1125	-0.0125	1	0.02

With 'load' F_j = 1 and 'load' F_k = 1 the responses will be:

Linear = 1.005	Quad = 1.01	Product = 1.1	Cubic = 1.02

Obviously the transformation from the "multi-valued" {b_{ij}, $c_{ij.j}$, $c_{ij.k}$, d_{ij}} terms to the "invariant" terms {$b1_{ij}$, $b2_{ij}$, $c1_{ij.j}$, $c2_{ij.j}$, $c3_{ij.k}$, $c4_{ij.k}$, $c5_{ij.k}$, $c6_{ij.k}$, $d1_{ij}$, $d2_{ij}$} can be reversed, provided that ALL (2 or 4) values of the "invariant" terms are defined.

LINEAR		LOAD SQUARED		2-LOAD PRODUCT				LOAD CUBED	
$b1_{ij}$	$b2_{ij}$	$c1_{ij.j}$	$c2_{ij.j}$	$c3_{ij.k}$	$c4_{ij.k}$	$c5_{ij.k}$	$c6_{ij.k}$	$d1_{ij}$	$d2_{ij}$
F_j	$\|F_j\|$	F_j*F_j	$F_j*\|F_j\|$	F_j*F_k	$\|F_j*F_k\|$	$F_j*\|F_k\|$	$\|F_j\|*F_k$	$F_j*F_j*F_j$	$\|F_j\|*\|F_j\|*\|F_j\|$
1	0.005	1	0.01	0.9625	0.0375	0.1125	-0.0125	1	0.02

transforms back to:

LINEAR		LOAD SQUARED		2-LOAD PRODUCT				LOAD CUBED	
b_{ij}		$c_{ij.j}$		$c_{ij.k}$				d_{ij}	
+	-	+	-	+ +	- -	+ -	- +	+	-
1.005	0.995	1.01	0.99	1.1	0.9	0.8	1.05	1.02	0.98